

Meaning Negotiation as Inference

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Abstract Meaning negotiation (MN) is the general process with which agents reach an agreement about the meaning of a set of terms. Artificial Intelligence scholars have dealt with the problem of MN by means of argumentations schemes, beliefs merging and information fusion operators, and ontology alignment but the proposed approaches depend upon the number of participants. In this paper, we give a general model of MN for an arbitrary number of agents, in which each participant discusses with the others her viewpoint by exhibiting it in an actual set of constraints on the meaning of the negotiated terms. We call this presentation of individual viewpoints an angle. The agents do not aim at forming a common viewpoint but, instead, at agreeing about an acceptable common angle. We analyze separately the process of MN by two agents (*bilateral* or *pairwise* MN) and by more than two agents (*multiparty* MN), and we use game theoretic models to understand how the process develops in both cases: the models are Bargaining Game for bilateral MN and English Auction for multiparty MN. We formalize the process of reaching such an agreement by giving a deduction system that comprises of rules that are consistent and adequate for representing MN.

Keywords Meaning negotiation · Agreement · Disagreement · Deduction · Viewpoints

1 Introduction and Motivations

In recent years, it has become clear that computer systems do not work in isolation. Rather, computer systems are increasingly acting as elements in a complex, distributed community of people and systems, which, in order to fulfill their tasks, must cooperate, coordinate their activities and communicate with each other. In fact, cooperation and coordination are needed almost everywhere computers are used. Relevant examples

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include health institutions, electricity networks, electronic commerce, robotic systems, digital libraries, military units etc.

Problems of coordination and cooperation are not a novelty due to the birth of automated systems. They exist at multiple levels of activity in a wide range of human agents as well. People achieve their own goals through communication and cooperation with other people; and, in industrial systems, with machines as well.

The main difficulty in agent cooperation and communication is to understand each other. People and, in general, *intelligent agents* come from different organizations and individuals and thus they have different backgrounds and, maybe, different expression languages. However, natural agents get to agreements as a means for solving conflicts. Consequently, artificial agents, in order to be reasonably similar to human agents, to an extent that grants their usefulness, are to be designed as agents that discuss to reach an agreement by starting from distinct viewpoints.

Intelligent agents have been considered in a wide number of reasons and applications, that is in all the situations in which people can delegate their interests to somebody else. In fact the word *intelligent* refers to the ability to behave, to reason and to perceive situations and the environment the agents are in like humans do. In all the applications of intelligent agents, a basic mechanism of agreement is required: information agents, electronic commerce agents, agents in e-learning systems and legal reasoning have to know the meaning of all the information they receive from others. In all the situations in which a misunderstanding arises, the system does not work as the user's expectations and it produces wrong outcomes.

To achieve an agreement there are fundamentally four possibilities:

- *delegate decision*: consisting in the choice of an external agent that decides for all the other agents involved;
- *judge decision*: consisting in the choice of an external agent that chooses among the proposals of the agents;
- *merging*: consisting in generating a new theory starting from the merged ones;
- *negotiation*: consisting in a sequence of actions aiming at the definition of a novel, shared position, emerged from the discussion itself, by means of a group of mechanisms, established as rules of the negotiation.

In general, negotiation is a dialogue between two or more agents by which they try to reach an agreement about something starting from different viewpoints about the shared object. A negotiation process is *quantitative* when the agents discuss about how to share a set of countable objects, whereas in *Meaning Negotiation*, on which we focus in this paper, the proposals are pieces of knowledge represented by terms, i.e. the expressions of what an agent knows about the negotiated terms. These pieces of knowledge may be accepted or rejected by the other discussants.

More specifically, Meaning Negotiation (henceforth MN) is a negotiation process in which the sharing object is the meaning of a set of terms. A common LucaWhy “acceptable”? A definition is a definition. Maybe replace with “common”, “standard”,... definition of MN is: the process that takes place when the involved agents have some knowledge (some data or information) to share but do not agree on what knowledge agents share and (possibly) how they reach an agreement about it.

In this paper, we focus on the processes that take place when the agents who negotiate agree about the mechanisms to reach an agreement and disagree about the meaning of the negotiated terms. In this case, agents *know* how to reach an agreement, and see the mechanisms themselves as a protocol. We aim at designing a model of

an inference engine whose derivations are indeed agreement processes. This approach views meaning negotiation as an inference process.

To clarify what a negotiation process indeed is, let us introduce an example that we will also employ in the rest of the paper as a running example.

Example 1 Consider two agents Alice and Bob that negotiate the meaning of the term “vehicle”. Suppose that Alice thinks that a vehicle always has two, three, four or six wheels; a handlebar or a steering wheel; a motor, or two or four bicycle pedals, or a tow bar. On the other hand, Bob thinks that it always has two, three or four wheels; a handlebar or a steering wheel; a motor, or two or four bicycle pedals. Alice and Bob are in disagreement because Bob does not know if a vehicle has a tow bar or not. \square

In this example, the MN depends upon the relevance of the terms the agents use. Alice and Bob define “vehicle” in different ways and with different terms. In fact, Alice uses “tow bar” and Bob does not. Bob does not say anything about the tow bar maybe he does not know what a tow bar is, or maybe he does not consider as relevant the properties about the tow bar. In this paper, we assume that the agents make assertions only about the properties they consider as relevant.

One of the main parameters of the MN is the number of the involved agents. In Section 1.1, we discuss in detail how a negotiation process takes place between two parties, whilst in Section 1.2 we discuss the situation arising when the number of involved partners is higher than two.

Before we do so, we further exemplify the specific problems of the definition of an inference engine, by considering a case taken from a common situation. We consider the Description Logic framework, where the notion of term, and definitions, in particular, of term, is part of the structure itself of the Logic. More specifically, in description logic, the *acceptability* of a concept is tested by the subsumption relation. Having two concepts A and B , A subsumes B ($A \sqsubseteq B$) when the definition for B is also a definition for A but not vice versa. Two concepts are equivalently defined when $A \sqsubseteq B$ and $B \sqsubseteq A$. Suppose that Alice considers A and B , where $A \sqsubseteq B$, as two plausible definitions for a concept X . Alice can accept a new definition C for X iff $A \sqsubseteq C \sqsubseteq B$ because Alice has a pair of concepts (A, B) describing the same things that C describes.

In the same way of the above mentioned Description Logic Framework, a logical formula φ is acceptable with respect to an agent when she shares the interpretation of all its terms. Therefore Alice always accepts φ when it is equivalent to her current angle α , that is $\alpha \leftrightarrow \varphi$ because $\mathcal{I}(\alpha) = \mathcal{I}(\varphi)$ where \mathcal{I} is an interpretation function, but Alice will accept φ also when she has a pair of feasible angles α and β such that $\alpha \rightarrow \varphi \rightarrow \beta$ because $\mathcal{I}(\alpha) \subseteq \mathcal{I}(\varphi) \subseteq \mathcal{I}(\beta)$. In the last case Alice shares the interpretation of all the terms in φ by means of two angles α and β that are partial representations of her knowledge she considers as acceptable.

An agent always accepts a logical theory when it is equivalent to her own one, i.e. when the two theories have the same set of semantical models. The logical equivalence is always a condition of agreement but there are situations in which it is not a necessary condition and some weaker conditions are sufficient to claim the agreement. In fact, a logical theory may be considered acceptable when it is a good compromise for an agent, i.e. the logical theory expresses an acceptable part of what the agent wants to express. In this sense, the set of semantical models of the proposal is not equal to the set of semantical models of the theory of the agent, but the agent has a superset and a subset of models bounding it.

Conclusively, the participants to a discussion may disagree, in fact, in three different ways:

- The properties used to define the terms are inconsistent and contradictory.
- The relevant properties for an agent are more/less than those expressed by one or more other agents.
- Some agents do not know the properties used by someone in the multiple agent system.

The idea of our framework is that the knowledge of an agent represents her viewpoint about the world and in order to negotiate with the other agents, an agent possibly has a set of acceptable portions of her knowledge that she may consider as good compromises with respect to her viewpoint.

We call *angle* any partial representation of a viewpoint. The knowledge of a negotiation is built by a single viewpoint and many angles, i.e. many partial representations of it. Moreover, in this paper we assume that angles are presented as *logical theories*, and in particular *propositional* ones. At the beginning of a MN process, agents are in disagreement, i.e. they have mutually inconsistent knowledge. By MN, they try to reach a common angle representing a shared acceptable knowledge, where the MN ends in positive way when the agents have a common knowledge, and it ends in negative way otherwise: agents are in *agreement* when they have found a set of constraints on the meaning of the negotiated terms that is accepted by both agents (this new theory is named, here, a *common angle*); *disagreement* when they are not in agreement. To negotiate the meaning of a set of terms means to propose definitions, properties, typical memberships of the terms' definitions, and/or to accept or to reject definitions.

1.1 Bilateral Meaning Negotiation

When the negotiation involves two agents, each proposal has one sender and one receiver. Game Theory scholars have dealt with several bilateral negotiation protocols like *Divorce* (Wurman et al, 2001), *Pleadings* (Gordon, 1993) and the *Bargaining Game* (Kambe, 1995). When the negotiation is bilateral the agents are called buyer and seller (which is typically the first proposing agent). Both the buyer and the seller have the same feasible actions: they make a proposal or accept or reject an offer.

As a matter of fact, it is the Bargaining game that offers the most natural framework for meaning negotiation, due to the need for negotiators to avoid the meaning of terms to be negotiated to be a *compromise* between the definitions preferred by the two agents, that implies that the negotiators play by moving themselves to the other's viewpoint with the maximally possible flexibility.

In the Bargaining Game, two agents have to share, say, one dollar and do this by each making a proposal. If the sum of their demands is less than one, they share the dollar, otherwise they have to make a new demand. The Bargaining Game is built by two stages:

- *Demand stage*: agents make a proposal and if the proposals are compatible, the negotiation ends in positive way; otherwise the second stage begins.
- *War of attrition*: agents have incompatible viewpoints and perform new demands. If the demands are compatible, the process ends positively, otherwise they make new ones.

In the Bargaining Game, players have a *negotiation power* that represents how often an agent cedes during the negotiation and how much she resists about her current angle. The negotiation power of an agent is captured by a set of partially ordered angles of her viewpoint. The partial order among the angles allows an agent to choose the next proposal to perform, and to evaluate the acceptability of the received offers. Moreover, the set of partially ordered angles has a minimum that identifies the last offer an agent proposes in a negotiation. We say that each agent has

- one single *stubborn* and
- many *flexible* angles

that are respectively the limit proposal (i.e., the last offer) and the acceptable ones, where each flexible angle is consistent with the stubborn knowledge.

The Meaning Negotiation process ends in a positive way (agreement) when both agents agree about a common definition of the meaning of the set of terms, i.e. they propose the same thing, or in a negative way (disagreement) when they are not in agreement and they have no more proposal to perform.

1.2 Multiple parties' Meaning Negotiation

When the number of agents is more than two, the negotiation is multiparty and each proposal has one sender and many receivers. A proposal may be accepted or rejected by all the agents or by only some of them, and the receivers may answer in different ways. The negotiation process for multiparty scenarios is computationally harder than the bilateral one and it needs the organization of the order in which agents make assertions (proposals, acceptance or rejection of offers) during the process. The modeling of the Meaning Negotiation in this case depends also on the role of the involved agents. Having $n + 1$ agents in the negotiation, the possible role distinctions are:

- 1- n : one seller and many buyers;
- n -1: many sellers and one buyer;
- n_1 - n_2 : many sellers and many buyers.

In the first case, the agents behave like in an auction. Before entering the auction, the seller establishes a maximal price for the item. The seller begins the game by making the initial request that is the *reservation price*. The auction develops by *beats*. A beat consists of:

1. the seller makes a request;
2. each buyer proposes a counteroffer or accepts the seller's proposal.

No more beat begins if the maximal price is reached or if the buying agents do not make new proposals. In an auction scenario, a proposal is also called a *bid*. The end of the auction is established by the seller, i.e. by the auctioneer. In general, in an auction there is only one winner, i.e. only one agent buys the item in the auction.

The second case, n -1, is similar to the first one. The sellers have to convince the buyer to accept the price they propose and when the buyer is not convinced she has to respond with another offer. In the Meaning Negotiation perspective, a buyer is not different from a seller because they have the same feasible action: accept an offer, reject an offer or make a proposal. Even if the agents generally have different strategies depending on their role, i.e. typically a buyer enhances instead the seller fall the last

offer, the purpose of the seller and of the buyer is the same: to meet the opponent's request. Therefore, buyers and sellers make new proposals in the same way, that is by *ceding* their last one.

The third case, n_1 - n_2 , is called *fish market*. For a reference on the different ways to perform negotiation, see Lomuscio et al (2003). It is not possible to make a modeling of the MN of this multiple-agent system structure because there is no agent monitoring the process and no behavioral guidelines for the players. In the first two cases, the auctioneers, the seller in the first and the buyer in the second, are the agents who control the Meaning Negotiation process and check whether an agreement is reached between the involved agents. As in the auction game, in the fish market each agent makes a proposal or accepts/rejects the opponents' one but there is no coordination among the agents. It may be the case that two or more agents make proposals simultaneously so each agent is a buyer, i.e. she makes an offer, and a seller, i.e. she evaluates the received offers, at the same time. The result is that a common proposal is difficult to find. In the worst case, where there are n agents involved in negotiation in total, this means there can be up to $n(n-1)/2$ negotiation threads. Clearly, from an analysis point of view, this makes such negotiation hard to handle.

In this paper, as in the main approach in the current Artificial Intelligence literature, we model the multiparty MN by reserving an agent, typically the first bidding one, to be the referee of the process and the game used to represent it is the auction. In Game Theory, there are several auction types (Benameur et al, 2002): English, Dutch, Vickrey, First-price sealed-bid etc. The types of auction differ on the behavior of the agents involved and on the number of the proposals the agents make. In this paper we use the English Auction because the agents behave as in the Bargaining Game. The English Auction Game begins by the proposal of the auctioneer that is called *reservation price* and it is the minimum price the agents have to pay to win the auction. In the next step of the English Auction, each player makes her offer by incrementing the last bidden one, i.e. the auctioneer's proposal. There is not a fixed number of turns for agents' bidding, instead the game continues until no more bids are performed. The game ends with a winner that is the agent who bids the highest offer.

In a MN perspective, the English Auction game is slightly different in the outcome. The goal of the negotiation is agents in sharing a viewpoint. Therefore the positive ending condition of the game is that all the agents make the same bid and the bidden proposal is the representation of their viewpoints.

There are MN contexts in which it is sufficient to have a "major" part of agreeing agents to consider positive the negotiation. In general "major part" means that a number of agents, typically more then 50% , but it may mean that a part of the most trustworthy agents are in agreement. In the former case, the minimum number of agreeing agents is a parameter of the game: suppose α is the chosen number for "major part", the MN continues until at least α agents agree about a common angle. The minimum number of agreeing agents is called *degree of sharing*. A MN process for more than two agents, say n agents, has two positive ending conditions and two types of positive outcomes, if a positive outcome exists:

- partially positive* : when the degree of sharing is less than the number of the participants ($\alpha < n$);
- totally positive* : when the degree of sharing is equal to the number of the negotiating agents ($\alpha = n$).

The latter case prevails when there are specialists about the negotiation subject into the multiple agent system and their opinion is more relevant than the opponents' ones. When participating to a negotiation process, the agents assume a viewpoint and many admissible angles of it. A specialist knows more about the negotiation subject than a less expert agent and her negotiation behavior will be to make concessions as few as possible. Conversely, if a no expert agent knows that an agent in the MAS is a specialist, then she trusts the specialist and probably makes concessions with respect to the proposals of the specialist. The degree of knowledge of an agent translates into the trustworthiness with respect to herself. In this paper, the trustworthiness of the agent is not specifically considered because it is out of the scope of the paper and it is left as a future work.

The role of the auctioneer is to monitor the game in order to understand when it ends and whether in a positive or a negative way. In general, the auctioneer is the first bidding agent but in a negotiation perspective she may play in two ways: active or passive. An active referee is a participant of the negotiation and the reservation price is her viewpoint. Moreover, an active referee makes herself proposals during the auction as all the other agents and she is considered in the agreement test. A passive auctioneer does not affect the negotiation. She only tests the process and makes only one bid, the first one for the reservation price.

1.3 Aims of proposed approach

The aim of the paper is to give a general model to represent the process of MN by means of a deduction system. Our formalization is based upon Game Theory notions of behavior of the agents during a negotiation/litigation. The negotiation process has already been dealt in terms of games but, to the best of our knowledge, only quantitative negotiation were studied. MN is not quantitative thus one of the main problem in dealing with it is the identification of the agreement and disagreement situations, i.e. the mutual evaluation of the proposals of the players. The purpose of the paper is to extend the current literature with the formalization of the MN problem by means of a deduction system that is independent of the number of the involved agents and of their expression languages. Our work begins with the study of the representation of the knowledge of the agents in a MN, and in particular the representation of the properties the agents consider as necessary and unforgivable in defining the meaning of the set of terms they are negotiating and, vice versa, which are the facultative ones, because these properties identify the negotiation space between agents. We call the first one the stubborn knowledge of the agent and flexible knowledge the second one.

The *first contribution* of the paper is the definition of the meaning negotiating agent in terms of her stubborn and flexible knowledge.

The *second contribution* of the paper is the study of the agreement and disagreement situations between the agents and the definition of the different ways in which they may be in disagreement (absolute, relative, essence and compatibility).

As said above, one important issue in MN is the evaluation of a received proposal. Agents make proposal and evaluate the opponents' one. The evaluation mechanism is not trivial when the negotiation is not quantitative. When is one definition of a set of terms better than another one? When are two or more definitions equivalent? Here, we study how a proposal is evaluated with respect to the knowledge of an agent. In our

model, the types of disagreement depend upon the relation among the proposal p and the stubborn and the flexible knowledge of the agent i who receives and evaluates p :

- *Call-away* occurs when p is a generalization¹ of the stubborn knowledge of i , thus it would correspond to dropping out some unquestionable knowledge.
- *Absolute disagreement* occurs when the stubborn knowledge of i is inconsistent with respect to p .
- *Essence disagreement* occurs when the flexible knowledge of i is inconsistent with respect to p .
- *Compatibility* occurs when p is consistent with the flexible knowledge of i but it is not a generalization or a restriction of i 's viewpoint.
- *Relative disagreement* occurs when p is a generalization of the flexible knowledge of i .

The call-away situations arise when an agent does not accept all the necessary requests of the other one and thus exits the MN so that the MN ends negatively.

An important point in MN as well as of the Multiple Agent Systems (MAS), is the strategical component in the definition of negotiating agent. In this paper we do not give any definition of strategy of agents but we assume that whenever an agent has to choose the next move, she has a way to do it. In general, in MAS literature there are two main ways in which the agents behave: collaborative and competitive. A collaborative agent always chooses the move that improves the welfare of the MAS she is in, whereas a competitive agent moves in order to achieve her goals and, possibly, to prevent the other ones. The study of the strategies in MN process needs the definition of MAS welfare and goals, and of the attainment of a goal.

The rest of the paper is organized as follows: in Section 2 we formalize the negotiating agents in terms of their knowledge and language, Section 3 defines the agreement and disagreement relations between agents and gives the deduction rules for bilateral and multiparty MN and Section 4 discusses the current approaches of Artificial Intelligence community for MN. The paper ends with the summary of the contributions of our work and with a discussion of future work (Section 5).

2 A Formalization of Negotiating Agents

We consider here a general MN process, so we abstract away from the particular terms whose meaning the agents are negotiating. We first consider the knowledge of negotiating agents (Section 2.1), i.e. what agents know about the meaning of the set of terms they are negotiating, and then their language (Section 2.2), i.e. how they represent their knowledge and how they make proposals during the MN.

2.1 The Knowledge of Negotiating Agents

When agents give the definition of a concept, they:

- give the necessary (*stubborn*) properties and the characterizing (*flexible*) ones;

¹ A theory A is a *generalization* of a theory B when the models of A are a superset of the models of B .

- give the properties that necessarily have not to hold and the ones that plausibly (flexibly) have not to hold; and
- give the formulas asserting what has not (stubbornly), or may not (flexibly), be used in the definition.

The notion of relevance of a formula is interesting at this stage of the definition, but instead of introducing a novel operator, we simply consider a formula as not relevant to an agent if she does not assert it. When i *asserts* a formula φ , she has a way to evaluate it: she thinks φ as positive or negative. If i does not assert φ then either i does not know φ , i.e., she is not able to evaluate it or i does not think φ is relevant in defining the negotiated meaning. So, we assume that *whenever i thinks φ as not relevant for the negotiation, i never asserts φ during the negotiation.*

Example 2 As in Example 1, consider the definition of the term “vehicle”. Alice (stubbornly) thinks that it always has two, three, four or six wheels; a handlebar or a steering wheel; a motor, or two or four bicycle pedals, or a tow bar. Moreover, Alice (flexibly) thinks that a “vehicle” may be defined only as a car, then having four wheels, a steering wheel, and a motor; or only as a bicycle, then having two wheels, a handlebar and two bicycle pedals. In other words, Alice has two acceptable ways to define a vehicle (namely, a car or a bicycle as particular “vehicles”) but she has only one general description of a “vehicle”. \square

The necessary and the characterizing properties of a concept definition are closely related to *EGG/YOLK* objects, introduced by Lehmann and Cohn (1994) as a way to represent class membership based on typicality of the members: the egg is the set of the class members and the yolk is the set of the *typical* ones. For instance, the class of “employees” of a company A may be defined as “the set of people that receive money from the company in exchange for carrying out the instructions of a person who is an employee of that company”, thus excluding, e.g., the head of the company (who has no boss), and the typical employee would include regular workers like secretaries and foremen. Another company B might have a different definition, e.g., including the head of the company, resulting in a mismatch. Nevertheless, if both companies provide some typical examples of “employees” it is possible that all of A ’s typical employees fit B ’s definition, and all of B ’s typical employees fit A ’s definition: $YOLK_B \leq EGG_A$ and $YOLK_A \leq EGG_B$, in the terminology of (Lehmann and Cohn, 1994).

In this paper, we use the same idea to express that negotiating agents have a *preference* over their knowledge: the properties an agent thinks as necessary are the typical ones, and the characterizing properties are those that are not typical but plausible. We focus on the models of the knowledge of an agent. The stubborn properties of a concept definition are the most acceptable ones, therefore they thus have more elements satisfying them than the flexible properties have. Hence, we represent the elements satisfying the stubborn properties in the egg and those satisfying the flexible ones in the yolk. Differently from the original model, concept definitions are here restricted by stubborn properties to the largest acceptable set of models, hence represented by the egg, whilst the yolk is employed to denote the most restricted knowledge, that is, the one on which the agents are flexible.

The stubborn properties never change during the negotiation; therefore, the egg is fixed at the beginning of the MN. Instead, the flexible part of the definition of a concept is the core of the proposal of a negotiating agent. Each proposal differs from the further ones in two possible ways: it may give a definition of the negotiated object

that is more descriptive than the next ones, or the given definition specifies properties that the next ones do not and vice versa. In the former case, we say that the agent carries out a *weakening action*, in the latter the agent carries out a *changing theory action*. In this paper, we do not consider how and why an agent chooses the next action to perform, but a general approach for dealing with agency in *multiple agent system (MAS)* is based on the representation of the choice of the action to perform by *attitudes*, which “are driving forces behind the actions of agents” (Meyer et al, 1999). In other words, attitudes are *the representation of the reasons that guide the agents in their behavior*. They are preferences between the criteria used to evaluate the feasible actions. In general, the main criteria for evaluating an action are:

1. The MAS welfare: is the action positive for all the agents in the MAS?
2. The personal advantage: is the action individually positive for the agent in choosing an action?

By attitude, we mean the *preference order of the evaluation criteria*. Following the enumeration in the list above, the main attitudes in agency are:

- *collaborative*: the main goal of the agent is the welfare of the MAS: 1 is preferred to 2;
- *competitive*: the action performed by a competitive agent are advantageous or not damaging herself: 2 is preferred to 1.

In a MN perspective, a collaborative agent aims at ending the process as soon as possible, whilst a competitive agent tends to stay as close as possible to her initial viewpoint. The collaborative and the competitive attitudes are dual.

However, none of the weakening or changing theory actions can be carried out with respect to a proposal if the proposal describes the necessary properties of the object in the MN. We say that in such a situation the agents always make a *stubbornness action* that is equivalent to *no more change*.

2.2 The Language of Negotiating Agents

Each agent i is represented by her language L_i , which is composed of two disjoint sublanguages (where we intend, with “language”, the set of well-formed formulae of a logical language):

- a *stubbornness* language containing the properties i deems as necessary in defining the negotiated meaning and
- a *flexible* language containing the properties i deems as not necessary in the MN.

Definition 1 (Σ_i and L_i) Consider an abstract set of terms and let Ag be the set of the *negotiating agents*. The *signature* Σ_i of an agent $i \in Ag$ is the pair $\langle \mathcal{P}_i, \alpha_i \rangle$ where

- \mathcal{P}_i is the set of the predicate symbols;
- $\alpha_i : \mathcal{P}_i \rightarrow \mathbb{N}$ is the arity function for predicate symbols.

The *language* L_i of $i \in Ag$ comprises of Σ_i -formulas defined inductively as follows:

- If $P \in \mathcal{P}_i$, $\alpha_i(P) = n$ and t_1, \dots, t_n are terms then $P(t_1, \dots, t_n)$ is a Σ_i -formula.
- If φ and ψ are Σ_i -formulas then $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \rightarrow \psi$ are Σ_i -formulas.




$\frac{flex_i^k \rightarrow flex_i^{k+1} \quad \neg(stub_i \leftrightarrow flex_i^k)}{flex_i^{k+1}} (W)$	
$\frac{flex_i^k \quad \neg(stub_i \leftrightarrow flex_i^k) \quad \neg(flex_i^k \rightarrow flex_i^{k+1}) \quad \neg(flex_i^{k+1} \rightarrow flex_i^k)}{flex_i^{k+1}} (C)$	
$\frac{\varphi \quad stub_i \leftrightarrow \varphi}{\varphi} (S)$	

Table 1 Rules for making new proposals and the corresponding EGG/YOLKs. The dark gray yolk identifies $flex_i^{k+1}$ and the light gray one identifies $flex_i^k$.

Definition 2 (Stubbornness and Flexibility of an agent) The agent i considers the formulas in L_i in two ways: *stubborn* or *flexible*. That is, the language L_i is divided in two disjoint sets: $L_i = L_{S_i} \cup L_{F_i}$, where

L_{S_i} is the set of stubborn formulas;

L_{F_i} is the set of flexible formulas.

We further define

$$stub_i = \bigwedge_{\varphi \in L_{S_i}} \varphi$$

and

$$flex_i = \bigwedge_{\varphi \in L_{F_i}} \varphi$$

During a negotiation process, the viewpoint of each agent is presented in a specific *angle*. In other words, a viewpoint is a hierarchy of theories, related by the partial order relation of weakening, and an element of this hierarchy is an angle. Each agent presents angles in sequence during the negotiation. Thus we call *current angle formula (CAF)* the angle presented at the current stage of the negotiation. A flexible formula $flex_i^k$ expresses the k^{th} angle discussed in the MN by the agent i and it changes during the process. We assume here that for each CAF $flex_i^k$ there is a stubborn formula in L_{S_i} that is a generalization of it. In general, during a negotiation of the meaning of a term, the agents relax their viewpoint in order to meet the opponent's one, and they do this only if the relaxing formula is not too general. Then, for each assertion in the MN, the agents have a maximal generalization of it and this is a formula in the stubbornness set. For instance, if the object of the negotiation is the meaning of *pen*, an agent is flexible on the ink color of the object but not on the fact that the object contains ink; then, the *red ink* predicate is a flexible one and the *contains ink* predicate is a stubborn one.

$flex_i^k$ changes during the MN by applying to it one of the rules for making new proposals given in Table 1: weakening (W), changing theory (C) or stubbornness (S). The EGG/YOLK representations show the collocation of the new proposal (in the stubbornness situation the new proposal is the same as the last one).

Let $flex_i^k$ be the last proposal of an agent i during a MN. There are two ways for i to make a new proposal $flex_i^{k+1}$. The weakening rule (W) states that i can propose $flex_i^{k+1}$ if $flex_i^{k+1}$ is entailed by $flex_i^k$ (i.e., $flex_i^k \rightarrow flex_i^{k+1}$) and $flex_i^k$ is not the most general formula the agent can negotiate (corresponding to her stubbornness viewpoint,

i.e., $flex_i^k \leftrightarrow stub_i$). Note that if i weakens, say, $flex_i^0$ to the new CAF $flex_i^1$, then i may be no more able to satisfy $flex_i^0$.

The rule (C) states that i can just change angle. Suppose that $flex_i^k$ is the last proposal of an agent i during a MN. There are two ways for i to make a new proposal $flex_i^{k+1}$. In the first case, expressed by the weakening rule (W), i proposes $flex_i^{k+1}$ if $flex_i^{k+1}$ is entailed by $flex_i^k$ (i.e., $flex_i^k \rightarrow flex_i^{k+1}$) and $flex_i^k$ is not the most general formula the agent can negotiate (corresponding to her stubbornness viewpoint, i.e., $flex_i^k \leftrightarrow stub_i$). In the second case, expressed by the rule (C), i just changes theory. Although we do not consider MN strategies in detail here, in general, an agent chooses whether to perform a weakening or a changing theory action by applying the corresponding rule, but there are situations in which one action is better than the other. For instance, when an agent checks the compatibility situation it seems better to weaken the theory rather than changing it so to try to entail the opponent's viewpoint, while in essence disagreement situations it seems better to change the theory rather than weakening it so to try to meet the opponent's viewpoint.

If agent i is in stubbornness does she continue the negotiation or does she have to exit it? We assume that the agent exits the MN only if all the agents in the negotiation are stubborn. But an agent does not know the opponent's stubbornness viewpoint, so the exit condition is recognized only by the system. However, the stubborn agent always makes the same proposal during the MN, as expressed by the rule (S). If $flex_i^k \leftrightarrow stub_i$ then $flex_i^{k_1} = flex_i^{k_1+1}$ for all $k_1 > k$.

Let us now go deeply inside the negotiation process constraints. If an agent i makes a weakening of $flex_i^0$ and has $flex_i^1$ as the CAF, then i is no more able to satisfy $flex_i^0$. As we show below, the process of negotiation, means relaxing of individual hierarchies. In particular, based upon the reasoning above, $flex_i^k$ is the k^{th} angle of agent i .

We introduce a set of Σ_i -structures as agents change angles during the negotiation process and these viewpoints have to be satisfied in different structures. We thus define the semantical structure of a signature, which is built by a domain set and an interpretation function mapping predicate symbols into tuples of elements of the domain. We use a parameter k to denote the k^{th} structure of the k^{th} angle.

Definition 3 Given a signature $\Sigma_i = \langle \mathcal{P}_i, \alpha_i \rangle$, a Σ_i -structure \mathcal{A}_i is a pair $\langle \mathcal{D}_i, \mathcal{I}_i \rangle$ where the domain \mathcal{D}_i is a finite non-empty set and the interpretation function \mathcal{I}_i is such that $\mathcal{I}_i(P) \subseteq \mathcal{D}_i^n$ for all $P \in \mathcal{P}_i$ for which $\alpha(P) = n$.

We define the set of Σ_i -structures \mathcal{A}_i^k as $\mathcal{S}_i = \{\mathcal{A}_i^k \mid \mathcal{A}_i^k = \langle \mathcal{D}_i^k, \mathcal{I}_i^k \rangle\}$ where $\mathcal{D}_i^k \subseteq \mathcal{D}_i$ is the domain set with respect to agent i and, for all pairs $(\mathcal{I}_i^k, \mathcal{I}_i^{k+1})$, if the $(k+1)^{\text{th}}$ rule that agent i applied is:

- (W), then $\mathcal{I}_i^k(P) \subseteq \mathcal{I}_i^{k+1}(P)$ for all $P \in \mathcal{P}_i$;
- (C), then $\mathcal{I}_i^k(P) \neq \mathcal{I}_i^{k+1}(P)$, $\mathcal{I}_i^k(P) \not\subseteq \mathcal{I}_i^{k+1}(P)$ and $\mathcal{I}_i^{k+1}(P) \not\subseteq \mathcal{I}_i^k(P)$ for all $P \in \mathcal{P}_i$;
- (S), then $\mathcal{I}_i^k(P) = \mathcal{I}_i^{k+1}(P)$, for all $P \in \mathcal{P}_i$.

If φ and ψ are Σ_i -formulas then:

- $\mathcal{A}_i^k \models P(t_1, \dots, t_n)$ iff $(\mathcal{I}_i(t_1), \dots, \mathcal{I}_i(t_n)) \in \mathcal{I}_i(P)$, where $P \in \mathcal{P}_i$ and t_1, \dots, t_n are terms;
- $\mathcal{A}_i^k \models \neg\varphi$ iff $\mathcal{A}_i^k \not\models \varphi$;
- $\mathcal{A}_i^k \models \varphi \wedge \psi$ iff $\mathcal{A}_i^k \models \varphi$ and $\mathcal{A}_i^k \models \psi$;
- $\mathcal{A}_i^k \models \varphi \vee \psi$ iff $\mathcal{A}_i^k \models \varphi$ or $\mathcal{A}_i^k \models \psi$;
- $\mathcal{A}_i^k \models \varphi \rightarrow \psi$ iff $\mathcal{A}_i^k \models \psi$ or $\mathcal{A}_i^k \not\models \varphi$.

Example 3 Suppose Alice defines “vehicle” as in Example 1. Then

$$\begin{aligned} stub_A = & (has2wheels \vee has3wheels \vee has4wheels \vee has6wheels) \wedge \\ & (hasHandlebar \vee hasSteeringWheel) \wedge \\ & (hasMotor \vee has2bicyclePedals \vee has4bicyclePedals \vee hasTowBar) \end{aligned}$$

is the stubbornness part of Alice’s knowledge whose interpretation is $\mathcal{I}(stub_A) = \{\text{bicycle, tandem, motorbike, scooter, truck, car, trailer, chariot}\}$. Let

$$flex_A^k = has4wheels \wedge hasSteeringWheel \wedge (hasMotor \vee has2bicyclePedals)$$

be the CAF of Alice that it is not equivalent to her stubbornness knowledge and its interpretation is $\mathcal{I}(flex_A^k) = \{\text{car, truck}\} \subset \mathcal{I}(stub_A)$. Suppose Alice changes her CAF by means of a weakening action (W); then:

$$\begin{aligned} flex_A^{k+1} = & (has4wheels \vee has2wheels) \wedge (hasSteeringWheel \vee hasHandlebar) \wedge \\ & (hasMotor \vee has2bicyclePedals) \end{aligned}$$

The interpretation of $flex_A^{k+1}$ is $\mathcal{I}(flex_A^{k+1}) = \{\text{motorbike, scooter, car, truck}\} \subset \mathcal{I}(flex_A^k)$. Otherwise, suppose Alice changes her CAF by means of a changing theory action (C); then:

$$flex_A^{k+1} = has6wheels \wedge hasSteeringWheel \wedge (hasMotor \vee hasTowBar)$$

The interpretation of $flex_A^{k+1}$ is $\mathcal{I}(flex_A^{k+1}) = \{\text{truck, trailer}\}$ and $\mathcal{I}(flex_A^{k+1}) \not\subset \mathcal{I}(flex_A^k)$. \square

3 The MN Process

In this section, we formalize the MN process by defining the negotiation language (Section 3.1), i.e. how the agents send their proposals to the opponents, and the negotiation rules (Section 3.2) governing the development of the MN process, both for bilateral (Section 3.2.1) and 1-n negotiation (Section 3.2.2). We then (Section 3.3) show how the bilateral MN develops depending upon the relation between the stubborn knowledge of the agents. We do not show the development of the 1-n MN because it can be viewed as $n - 1$ instances of bilateral MN between the auctioneer and the other agents, where n is the number of the involved agents.

During the MN, agents make proposals and say if they are in agreement or not with respect to the proposals made by the opponents. Proposals are negotiation formulas like $j : \varphi$, where we assume that the opponent i is able to recognize the name label j in $j : \varphi$ and remove it in order to evaluate φ .

In general, negotiating agents may not share the same language but have different signatures. Hence, when i evaluates an assertion by j , she first has to *translate* the symbols occurring in it to symbols belonging to her signature. Such a translation depends, of course, on the particular terms that are being considered for the negotiation, so we assume abstractly that for each pair of agents (i, j) there is the *translation function* $\tau_{i,j}$ such that:

$$\tau_{i,j} : \Sigma_j \rightarrow \Sigma_i.$$

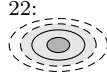


Fig. 1 The EGG/YOLK representation of the opponent's offer from agent i 's viewpoint identified by the plain lines, for call-away.

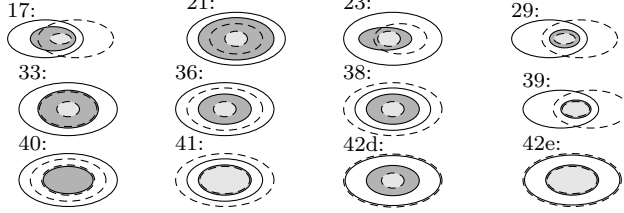


Fig. 2 The EGG/YOLK representation of the opponent's offer from agent i 's viewpoint identified by the plain lines, for agreement.

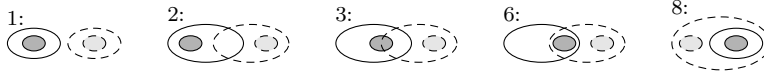


Fig. 3 The EGG/YOLK representation of the opponent's offer from agent i 's viewpoint identified by the plain lines, for absolute disagreement.

When j asserts φ (i.e., $j : \varphi$), i is not able to find which part of φ is in the stubbornness set of j , since she only knows that $\varphi = stub_j \wedge \psi^k$ where $stub_j$ is the conjunction of all the formulas in L_{S_j} and ψ^k is the k^{th} angle of j .

In the following, we describe the main conditions an agent has to test in order to evaluate the opponent proposal and to identify the negotiation condition she is in. We suppose that j is the first proponent (bidding) agent and that i is the agent evaluating j 's proposal. Figures 1, 2 3, 4, 5, and 6 show the EGG/YOLK representations in which i is identified by the plain line and j by the dashed line for each condition that i tests; the numbering is that of (Lehmann and Cohn, 1994). Let φ be the proposal of j . When the MN begins the agent receiving the first proposal controls that it is not too general and not too restrictive with respect to her viewpoint.

- If the received proposal φ is too general ($stub_i \rightarrow \tau_{i,j}(\varphi)$), then the agent i cannot negotiate with j because no generalization of her stubbornness knowledge is acceptable. In this case i thinks they are in *call-away* and the negotiation ends in a negative way. The corresponding EGG/YOLK representation is shown in Figure 1.
- Otherwise, in the case in which the received proposal φ is too restrictive ($\tau_{i,j}(\varphi) \rightarrow flex_i^0$) the only action i can perform is to re-initiate the MN by proposing her angle that is a generalization of φ .

When both of the previous cases are negative, the agent i evaluates how much acceptable is φ .

- The ideal situation is the agreement. As said before, a proposal is considered acceptable when it is equivalent to the current angle of i ($flex_i^k \leftrightarrow \tau_{i,j}(\varphi)$) or when it is representable by means of a pair of feasible angles, ($flex_i^k \rightarrow \tau_{i,j}(\varphi) \wedge (\tau_{i,j}(\varphi) \rightarrow flex_i^{k+1})$). In our formalism, if $flex_i^k \rightarrow \tau_{i,j}(\varphi)$ then there always exists $flex_i^{k+1}$ such that the previous condition is true because, as said in Section 2.2, for each

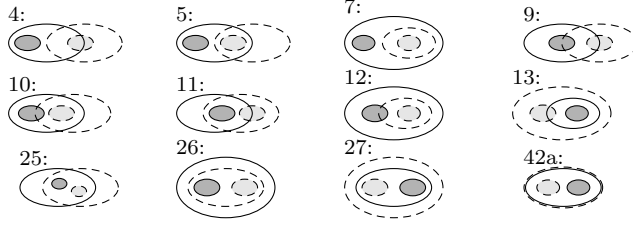


Fig. 4 The EGG/YOLK representation of the opponent's offer from agent i 's viewpoint identified by the plain lines, for essence disagreement.

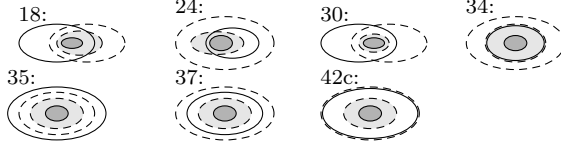


Fig. 5 The EGG/YOLK representation of the opponent's offer from agent i 's viewpoint identified by the plain lines, for relative disagreement.

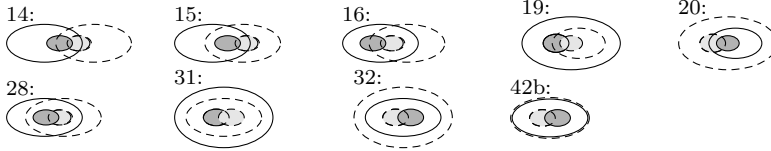


Fig. 6 The EGG/YOLK representation of the opponent's offer from agent i 's viewpoint identified by the plain lines, for compatibility.

$flex_i^k$ there is a stubborn formula in L_{S_i} that is a generalization of it. Thus, $flex_i^k \rightarrow \tau_{i,j}(\varphi)$ and $flex_i^k \rightarrow stub_i$ yield $\tau_{i,j}(\varphi) \rightarrow stub_i$. In fact, it is not possible that $stub_i \rightarrow \tau_{i,j}(\varphi)$ because this is the call-away condition. Thus, the sufficient condition to reach the agreement is $flex_i^k \rightarrow \tau_{i,j}(\varphi)$. The egg-yolk configurations for agreement are depicted in Figure 2. When agents are not in agreement, they disagree in many ways and i finds which type of disagreement is between φ and her current angle $flex_i^k$.

- If the proposal of j is not consistent with the stubbornness knowledge of agent i , $\neg(stub_i \wedge \tau_{i,j}(\varphi))$ then the agents are in absolute disagreement (Figure 3).
- If i and j are not in absolute disagreement, i 's CAF is consistent with respect to j 's proposal, and there is no generalization/restriction relation between $flex_i^k$ and φ , $\neg(flex_i^k \wedge \tau_{i,j}(\varphi)) \wedge (stub_i \vee \tau_{i,j}(\varphi))$, then the agents are in essence disagreement (Figure 4).
- If i and j are neither in essence nor in absolute disagreement and φ is a generalization of i 's CAF, $(flex_i^k \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^k)$, then the agents are in relative disagreement (Figure 5).
- If i and j are neither in absolute nor in relative disagreement, i 's CAF is consistent with respect to φ , and i 's CAF is not a weakening of φ , $(flex_i^k \vee \tau_{i,j}(\varphi)) \wedge \neg(flex_i^k \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^k)$, then the agents are in the compatibility relation (Figure 6).

After evaluating the received proposal, an agent informs the opponent about the negotiation situation she thinks to be in, in order to give to the opponent a motivation of the potential disagreement, i.e. the non acceptability of her proposal. To this end, we extend the formulas in the agent language:

Definition 4 (L_i extension) If φ is a proposal received by i in the negotiation process, then it is a formula asserted by somebody as $j : \varphi$, with $j \neq i$. We extend the language L_i with the formulas **absDis**($j : \varphi$), **essDis**($j : \varphi$), **relDis**($j : \varphi$), **comp**($j : \varphi$), and **agree**($j : \varphi$). For $\mathcal{A}_i^k = \langle \mathcal{D}_i^k, \mathcal{I}_i^k \rangle$ a Σ_i -structure, the semantics of these additional formulas is:

- $\mathcal{A}_i^k \models \mathbf{absDis}(j : \varphi)$ iff $\mathcal{A}_i^k \models \neg(stub_i \wedge \tau_{i,j}(\varphi))$;
- $\mathcal{A}_i^k \models \mathbf{essDis}(j : \varphi)$ iff $\mathcal{A}_i^k \models (stub_i \vee \tau_{i,j}(\varphi)) \wedge \neg(flex_i^k \wedge \tau_{i,j}(\varphi))$;
- $\mathcal{A}_i^k \models \mathbf{relDis}(j : \varphi)$ iff $\mathcal{A}_i^k \models (flex_i^k \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^k)$;
- $\mathcal{A}_i^k \models \mathbf{comp}(j : \varphi)$ iff $\mathcal{A}_i^k \models (flex_i^k \vee \tau_{i,j}(\varphi)) \wedge \neg(flex_i^k \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^k)$;
- $\mathcal{A}_i^k \models \mathbf{agree}(j : \varphi)$ iff $\mathcal{A}_i^k \models (flex_i^k \rightarrow \tau_{i,j}(\varphi))$.

We did not define a sentence **callAway**($j : \varphi$) as the call-away condition interrupts the MN. It is also important to remark that in our system we restrict the evaluation of agent proposals to formulas in the basic agent language, so no assertion can be made by agents using extended (and nested) formulas like **agree**(**comp**($j : \varphi$)). This restriction avoids nested MN processes.

In the two following subsections, we define the negotiation language and the deductive rules for the MN process.

3.1 MN language

The negotiation language L is built by the assertions of the agents during the negotiation, i.e., labeled formulas $i : \varphi$ meaning that agent $i \in \text{Ag}$ asserts the formula $\varphi \in L_i$. That is, $i : \varphi$ represents a proposal the agent i makes in the negotiation and typically represents her CAF or the evaluation of ψ asserted by j when φ is **R**($j : \psi$) where **R** is one of the disagreement relations **absDis**, **essDis**, **relDis**.

Definition 5 (Σ -formula) The *signature* of the MN language L is $\Sigma = \langle \mathcal{P}, \{\alpha_i\}_{i \in \text{Ag}} \rangle$ where $\mathcal{P} = \bigcup_{i \in \text{Ag}} \mathcal{P}_i$ and $\alpha_i : \mathcal{P}_i \rightarrow \mathbb{N}$ is the arity function for predicate symbols. Let φ be a L_i formula for some $i \in \text{Ag}$; then L comprises of Σ -formulas defined as follows:

- $i : \varphi$ is a Σ -formula;
- if φ_1 and φ_2 are Σ -formulas then $\varphi_1 \cap \varphi_2$ is a Σ -formula.

Let $\mathcal{N}^k = (\{\mathcal{A}_i^k\}_{i \in \text{Ag}, k \in \mathbb{N}}, \mathcal{F})$ be a Σ -structure where $\{\mathcal{A}_i^k\}_{i \in \text{Ag}, k \in \mathbb{N}}$ is the domain set and \mathcal{F} is an evaluation function that maps name labels into Ag . Then:

- $\mathcal{N}^k \models i : \varphi$ iff $\mathcal{A}_{\mathcal{F}(i)}^k \models \varphi$;
- $\mathcal{N}^k \models \varphi_1 \cap \varphi_2$ iff $\mathcal{N}^k \models \varphi_1$ and $\mathcal{N}^k \models \varphi_2$.

We need only the conjunction operation because during the negotiation, the stream of dialog begins with the proposal of the first bidding agent, that is the auctioneer in the $1 - n$ MN, and continues with pairs of offer evaluation and proposal of the following proposing agents.

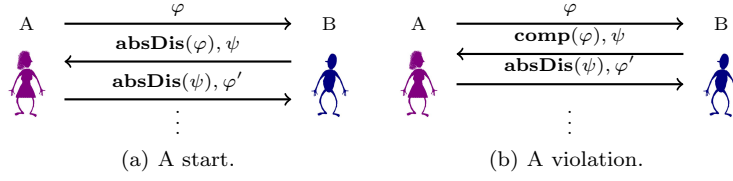


Fig. 7 Two bilateral MN scenarios.

$$\begin{array}{c}
 \frac{j : \varphi \quad \neg(stub_i \wedge \tau_{i,j}(\varphi))}{i : \mathbf{absDis}(j : \varphi) \cap i : flex_i^1} \quad (AD) \\
 \frac{j : \varphi \quad \neg(flex_i^0 \wedge \tau_{i,j}(\varphi)) \wedge (stub_i \vee \tau_{i,j}(\varphi))}{i : \mathbf{essDis}(j : \varphi) \cap i : flex_i^1} \quad (ED) \\
 \frac{j : \varphi \quad \neg(flex_i^0 \rightarrow \tau_{i,j}(\varphi)) \wedge (\tau_{i,j}(\varphi) \rightarrow flex_i^0)}{i : flex_i^0} \quad (I) \\
 \frac{j : \varphi \quad (flex_i^0 \rightarrow \tau_{i,j}(\varphi))}{i : \mathbf{agree}(j : \varphi) \cap i : \tau_{i,j}(\varphi)} \quad (Ag) \\
 \frac{j : \varphi \quad (flex_i^0 \vee \tau_{i,j}(\varphi)) \wedge \neg(flex_i^0 \rightarrow \tau_{i,j}(\varphi)) \wedge \neg(\tau_{i,j}(\varphi) \rightarrow flex_i^0)}{i : \mathbf{comp}(j : \varphi) \cap i : flex_i^1} \quad (Co)
 \end{array}$$

Table 2 Rules for the second proposing agent.

3.2 MN Rules

In this section, we provide the deductive rules for the MN process. We distinguish between pairwise MN and one-to-many MN because in the latter case an agent behaves differently when she is the auctioneer. Moreover, in a 1 – n MN the supervisor system ends the negotiation in a positive way when all or an acceptable part of the agents share a common angle: in the former case the negotiation is totally positive and in the latter it is partially positive.

3.2.1 MN Rules: 1-1 MN

We give the transition rules the agents use to negotiate depending on the mutual negotiation position they test and on their flexibility; these rules are coupled with those in Table 1. No rules are needed for the first proposing agent because she only makes a proposal; conversely the second and the following proposing agents make proposals and assert the evaluation of the received offer.

There are different rules for the second proposing agent and the following ones. The second proposing agent has to check if the first proposal is too general or too restrictive and thus if the negotiation has to end immediately in the former case (call-away relation), or to re-initiate with a new proposal in the latter case ((I) rule). Therefore, a following proposing agent receives proposals that are not restrictions of her initial one and she has only to test if a received proposal is not too general.

Consider the scenario in Figure 7(a): Alice (A) makes the proposal φ and Bob (B) evaluates it, where B 's reasoning is based upon two tests:

1. The relation between his CAF and φ . B 's CAF may be in agreement ($\varphi \leftrightarrow flex_B^k$) or not with φ , and B recognizes it by testing the condition listed above.
2. His stubbornness condition, i.e., if his CAF is $stub_B$ ($flex_B^k \leftrightarrow stub_B$) or not. Whenever B is stubborn, he performs the same counterproposal, otherwise he may relax his CAF by the (W) rule of Table 1 ($flex_B^k \rightarrow flex_B^{k+1}$) or change his theory by the (C) rule of Table 1 ($\neg(stub_i \leftrightarrow flex_i^k)$ and $\neg(flex_i^k \rightarrow flex_i^{k+1})$ and $\neg(flex_i^{k+1} \rightarrow flex_i^k)$).

At the end of his evaluation, B replies to A with a counterproposal ψ . When A evaluates ψ she has to consider the relation between her CAF and ψ , her stubbornness condition ($stub_A \leftrightarrow flex_A^k$) and B 's evaluation. The evaluation of the opponent agent helps agents in choosing the new proposal. The choice of the action, weakening or changing theory, and of the next proposal depends on the agent's attitude: a *collaborative agent* chooses the proposal that improves the negotiation relation with the opponent, whereas a *competitive agent* chooses the proposal that changes the least the relation with the opponent. For instance, if B says to A that when A proposes φ they are in essence disagreement, and B makes the proposal ψ , A will propose φ_1 or φ_2 , both inferred from φ by applying (W) or (C). When A is collaborative, she will propose φ_1 because she knows that they will be in agreement. Conversely, A will propose φ_2 , if A is competitive, because she knows that they will remain in essence disagreement.

Suppose B says to A that when A proposes φ they are in relative disagreement ($\psi \rightarrow \varphi$) and B makes the proposal ψ , then A knows that they are in agreement when she proposes ψ .

To support the interaction sketched above, we define the system MND to consist of the standard introduction and elimination rules for the connectives of L_i and L , and of two sets of rules: one set for the second proposing agent (Table 2) and another set for the following proposing agents (Table 3). For the sake of space, we omit the assumption of non call-away conditions in negotiation rules and explain only some of the rules by example.

Assume that A begins a MN by proposing $flex_A^0$ to B . B evaluates $\tau_{B,A}(flex_A^0)$ with respect to his initial angle $flex_B^0$ and suppose B thinks that $\tau_{B,A}(flex_A^0)$ is too strict, i.e., $\tau_{B,A}(flex_A^0) \rightarrow flex_B^0$. Thus, B cannot accept $\tau_{B,A}(flex_A^0)$ and re-initiates the MN by the rule (I) and proposes $flex_B^0$ by $B : flex_B^0$. Otherwise, suppose B thinks that $\tau_{B,A}(flex_A^0)$ is entailed by his initial angle $flex_B^0$ and that $\tau_{B,A}(flex_A^0)$ is not too general, i.e., it is not entailed by $stub_B$. In this case, B knows that A cannot accept $flex_B^0$ because it is too strict with respect to her viewpoint (explained in the beginning of Section 3), thus if B accepts $\tau_{B,A}(flex_A^0)$ by (Ag) because it satisfies the precondition ($flex_B^0 \rightarrow \tau_{B,A}(flex_A^0)$), and says $B : \mathbf{agree}(A : flex_A^0)$. This is the reason why there is no rule (RD) in Table 2 for relative disagreement relation. Consider the case in which B thinks that the proposal of A , $flex_A^0$, is consistent to his initial angle $flex_B^0$ by (Co). B says to A that they are in the compatibility relation by $B : \mathbf{comp}(A : flex_A^0)$ and makes a new proposal $B : flex_B^1$ such that $flex_B^0 \rightarrow flex_B^1$ (rule (W)). Now A thinks that $\tau_{A,B}(flex_B^1)$ is an acceptable angle of her initial viewpoint, i.e. $flex_A^1 \leftrightarrow \tau_{A,B}(flex_B^1)$. Thus A agrees with B and says $A : \mathbf{agree}(B : flex_B^1)$ by ($Co-Ag$). It may be the case that agents make proposals that become inconsistent with the received one. This inconsistency is tested by the opponent agent, not by the bidding one, because in MND

agents choose the new proposal only with respect to their angles and not with respect to the opponent's one.

Consider now the scenario in Figure 7(b). B evaluates the proposal of A , tests the compatibility relation, and makes the counterproposal. A evaluates it and finds they are inconsistent. In situations like this, agents perform proposals that violate the MN relation among agents; we call such a proposal a *violation* and the rule causing it a *violation rule*. In Table 3, the violation rules are $(ED-AD)$ and $(ED-Co)$.

The MN develops by agents making proposals and asserting if they are in agreement or not. The entire process is controlled by a supervisor, an external viewpoint, which tests if the MN ends and if the outcome is positive or negative. Table 4 shows the transition rules for the system. We write

- $j : \mathbf{na}(i : \varphi)$ to say that agent j thinks she is not in agreement with $i : \varphi$, and
- $*(i, j)$ to say *whatever the system state is* different from the final ones (*Agreement* and *Disagreement*), i.e., whether the system is in *Init* or *Negotiate*.

The MN begins when agents make proposals in turns $(i : \varphi, j : \psi)$ and they are not in agreement ($j : \mathbf{na}(i : \varphi)$) by (N) . The MN ends with a positive outcome (φ) when each agent agrees on a proposal ($j : \mathbf{agree}(i : \varphi)$), otherwise the MN ends with a negative outcome if there are no more proposals to perform ($stub_i \leftrightarrow \varphi$ and $stub_j \leftrightarrow \psi$) and agents do not agree on a common acceptable angle ($j : \mathbf{na}(i : \varphi)$).

Example 4 Let Alice and Bob be two agents negotiating the definition of the term “vehicle” as in Example 1. Suppose that the initial viewpoint of Alice is

$$flex_A^0 = has2wheels \wedge hasSteeringWheel \wedge (hasMotor \vee has2bicyclePedals)$$

and her stubbornness knowledge is as in Example 3. Suppose that Bob's initial viewpoint is

$$flex_B^0 = has2wheels \wedge hasHandlebar \wedge has2bicyclePedals$$

and his stubbornness knowledge is

$$\begin{aligned} stub_B = & (has2wheels \vee has3wheels \vee has4wheels) \wedge \\ & (hasHandlebar \vee hasSteeringWheel) \wedge \\ & (hasMotor \vee has2bicyclePedals \vee has4bicyclePedals) \end{aligned}$$

Alice is the first bidding agent and she proposes $flex_A^0$ to Bob, who receives the proposal and evaluates it. Bob tests that they are in compatibility because $(flex_B^0 \vee \tau_{B,A}(flex_A^0)) \wedge \neg(flex_B^0 \rightarrow \tau_{B,A}(flex_A^0)) \wedge \neg(\tau_{B,A}(flex_A^0) \rightarrow flex_B^0)$. Bob chooses the new CAF by a weakening action (W) in

$$\begin{aligned} flex_B^1 = & (has2wheels \vee has4wheels) \wedge \\ & (hasHandlebar \vee hasSteeringWheel) \wedge has2bicyclePedals \end{aligned}$$

Bob uses the (Co) rule and sends his CAF to Alice:

$$\frac{A : flex_A^0 \quad (flex_B^0 \vee \tau_{B,A}(flex_A^0)) \wedge \neg(flex_B^0 \rightarrow \tau_{B,A}(flex_A^0)) \wedge \neg(\tau_{B,A}(flex_A^0) \rightarrow flex_B^0)}{B : \mathbf{comp}(A : flex_A^0) \cap B : flex_B^1} \quad (Co)$$

The system continues the MN by:

$$\frac{*(A, B) \quad A : flex_A^0 \quad B : \mathbf{comp}(A : flex_A^0) \quad B : flex_B^1}{Negotiate(A, B)} \quad (N)$$

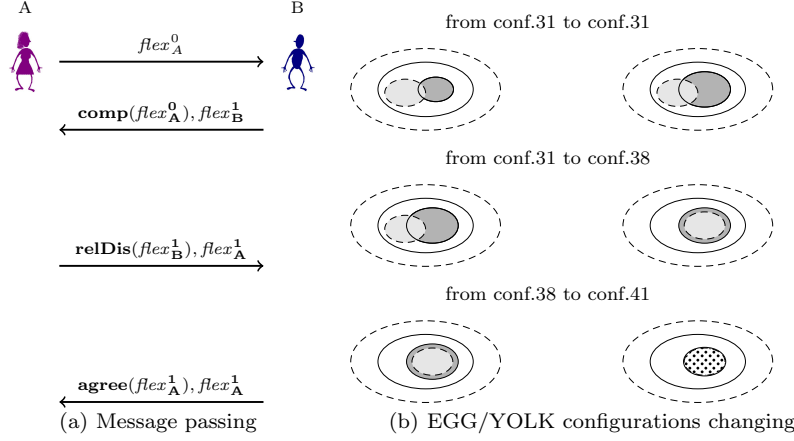


Fig. 8 The MN scenario of Example 4: the message passing flow (a) and the changes of the EGG/YOLKs of the agents (b). Alice is identified by plain lines and Bob by dashed lines. White yolks represent the precedent proposal of the agent and the dotted gray yolk is the positive outcome of the scenario.

Alice receives $flex_B^1$ and she has to make a weakening or a changing theory action because Bob did not say they were in agreement nor in relative disagreement. Alice performs a changing theory action by the rule (C) and her CAF is

$$flex_A^1 = has2wheels \wedge (hasHandlebar \vee hasSteeringWheel) \wedge has2bicyclePedals$$

Alice thinks they are in relative disagreement since $(flex_A^1 \rightarrow \tau_{A,B}(flex_B^1)) \wedge \neg(\tau_{A,B}(flex_B^1) \rightarrow flex_A^1)$, and she uses the rule (Co-RD) to inform Bob that they are in relative disagreement:

$$\frac{B : \mathbf{comp}(A : flex_A^0) \cap B : flex_B^1 \quad (flex_A^1 \rightarrow \tau_{A,B}(flex_B^1)) \wedge \neg(\tau_{A,B}(flex_B^1) \rightarrow flex_A^1)}{A : \mathbf{relDis}(B : flex_B^1) \cap A : flex_A^1} \text{ (Co-RD)}$$

The system continues the MN by:

$$\frac{*(B, A) \quad B : flex_B^1 \quad A : \mathbf{relDis}(B : flex_B^1) \quad A : flex_A^1}{Negotiate(B, A)} \text{ (N)}$$

Bob receives $flex_A^1$ and he accepts it because Alice said they are in relative disagreement.

$$\frac{A : \mathbf{relDis}(B : flex_B^1) \cap A : flex_A^1}{B : \mathbf{agree}(A : flex_A^1) \cap B : \tau_{B,A}(flex_A^1)} \text{ (RD-Ag)}$$

The system closes the MN by:

$$\frac{*(A, B) \quad A : flex_A^1 \quad B : \mathbf{agree}(A : flex_A^1)}{Agreement(A, B)} \text{ (A)}$$

with a positive outcome, $flex_A^1$.

Fig. 13 shows the message flow between Alice and Bob (Fig. 8(a)), and the changes of their EGG/YOLK configurations (Fig. 8(b)). \square

The classification of the agreement conditions provided above is complete, in the sense that there is no other possible configuration of EGG/YOLKs, as shown by Lehmann and Cohn in (1994). Based on the completeness of that analysis, we can show the following results.

Theorem 1 *MND for bilateral MN is consistent.*

Proof Consider two agents represented in the *MND* system with sets L_{S_1} and L_{S_2} of stubbornness formulas and sets L_{F_1} and L_{F_2} of flexible formulas. To prove that *MND* is consistent, we show that if a Σ_i formula ξ is inferred using the *MND* rules, or, in other terms, is deduced as a theorem in the system, then ξ represents a proposal that is acceptable by both agents. In other words, we aim at proving that when the rules yield ξ then ξ generalizes both L_{F_1} and L_{F_2} and is generalized by both L_{S_1} and L_{S_2} . To prove this, we need to show that:

(i) The rules for making new proposals yield a relation that is acceptable from the viewpoint of the agent who made the proposal before and infer a new proposal again still acceptable. In other terms, if an agent makes a proposal that is generalized by the set of stubbornness formulas L_{S_i} , and is a generalization of the set of flexible formulas L_{F_i} , for one agent, the rules infer a new proposal that is in the same relationships with L_{S_i} and L_{F_i} .

(ii) The rules for the second proposing agent infer the relation between the agents at that step of the negotiation.

(iii) The rules for the following proposing agent do the same as the rules for the second proposing agent, taking into account that this step takes place after the step of the second proposing agent.

(iv) The system transition rules close the negotiation only when the proposal is acceptable by both agents, namely generalizes both L_{F_i} and is generalized by both L_{S_i} sets.

Let us now consider a formula ξ that is acceptable by the two agents, and let us consider the rules that produce transitions in the system. In particular, if ξ is inferred by means of one of the rules *(AD)*, *(ED)*, *(I)*, *(Co)*, *(Ag)* for the second proposing agent, or by means of one of the rules given in Fig. 3 for the following proposing agent, then the possible results of the step described above are given by the application of the system transition rules. Evidently, if ξ is inferred, then the rule *(D)* does not apply. If *(N)* applies, and one more inference is performed, then the rules *(W)*, *(C)*, *(S)* allow us to infer a different formula. Suppose now, by contradiction, that the new formula ξ is not acceptable by one of the agents (in the sense that either is not a generalization of her set of flexible formulas or it is not generalized by the set of stubbornness formulas). As a consequence, one agent has called herself away, as we stated above. This, however, is impossible, by construction of the rules for the second and following proposals. Conversely, if the transition rule *(D)* applies and, therefore, the agents have incompatible viewpoints, then ξ is not inferred through the system, because it is not a generalization of both flexible sets of formulas and generalizes by both stubbornness sets of formulas. Clearly, by means of the full set of rules, it is not possible to do so when the agents have consistent viewpoints. \square

We say that a deductive system is adequate to represent a MN between two agents when it infers an outcome iff an agreement is reachable between the agents, otherwise it does not produce any result.

Definition 6 A deductive system \mathcal{R} is *adequate* to represent the MN process between two agents, i and j , when

$$\mathcal{R} \text{ infers } \begin{cases} \varphi & \text{iff for all } x \in \text{Ag there exists } k \in \mathbb{N} \text{ s.t. } \mathcal{A}_x^k \models (\text{flex}_x^k \rightarrow \varphi) \wedge (\varphi \rightarrow \text{stub}_x) \\ \perp & \text{otherwise} \end{cases}$$

where $\text{Ag} = \{i, j\}$ and $\varphi \in \bigcup_{x \in \text{Ag}} \mathbf{L}_x$.

Theorem 2 *MND is adequate to represent the MN of two agents.*

Proof We consider two agents that have consistent viewpoints, namely such that there exists a possible common angle. Their stubbornness sets and their flexible sets of formulas are in one of the EGG/YOLK configurations except number 1. Suppose now that the *MND* system infers a Σ_i formula ξ . Then, ξ is a common angle. Conversely, suppose that *MND* does not infer any Σ_i formula. Then, the agents are in call-away situation. Suppose now that the two agents have inconsistent viewpoints (configuration 1). The relation established is absolute disagreement. The result is that no formula can be inferred through the system, which is consistent by Theorem 1. Hence, overall, the system is adequate. \square

For MN processes that are built on finite signature theories, we obtain the following decidability result:

Corollary 1 *MN is decidable for theories with finite signature under the assumption of competitive agents.*

Proof Consider an MN between competitive agents on a language with finite signature. The number of possible proposals the agents can exchange during a MN process is formed by the possible formulas that can be built on the signature, which is finite. Since the rules of *MND* are finite and the new possible proposals are finite, and the number of applications of each rule is limited to the number of proposals the other negotiator can perform, the number of steps that will be performed, in any algorithmic solution to the problem, is finite as well. \square

In the following section, we extend the *MND* for 1-n MN in which one agent is the referee.

3.2.2 MN Rules: 1-n MN

When the Meaning Negotiation involves more than two agents, it may be viewed as an English Auction Game. An agent behaves differently if she is the auctioneer or not. The referee is the agent who receives all the proposals of the others and finds which one is shared by the agents. The auctioneer is a player himself; he makes a proposal at each new bid. The auctioneer replicates the same proposal to each of the negotiating agents. As a 1-1 MN player, the auctioneer evaluates each received proposal by testing the validity of the conditions listed above: he checks the relation between each received proposal and his stubbornness knowledge and his flexible one. An auctioneer differs from the other negotiating agents in the number of the evaluations he has to do. Moreover, when the auctioneer infers the next proposal to perform by the weakening or the changing rules, the proposal may be related in more ways than that of the proposal made by other agents. In fact, in 1-1 MN it is not possible to reach the absolute

disagreement by a relative disagreement situation because, when an agent i informs her opponent j that they are in relative disagreement, then j knows that i proposed one of i 's CAF that is a restriction of her CAF and j accepts it. Instead, in 1-n MN, the previous situations may raise: the auctioneer may not accept the proposal of one of the negotiating agents who said that they are in relative disagreement, because the proposal is not shared by the other agents. The set of deductive rules for the auctioneer is an extension of those in Table 3 with those in Table 5. In particular, all the added rules are violations and they represent the changing of the negotiation situation from relative disagreement to the relations of absolute disagreement, essence disagreement and compatibility, and from agreement to all the possible relations between agents: absolute disagreement, essence disagreement, compatibility, relative disagreement and agreement. The rule ($Ag-Ag$) is not only for the auctioneer but also for negotiating agents and it is used by them whenever the auctioneer proposes an acceptable angle.

Moreover, the system transition rules are different from the 1-1 MN ones (Table 4) because the agreement and disagreement conditions are different. In particular, the test of the agreement condition needs to count the number of agreeing agents. The 1-n MN ends in:

- *disagreement* when all the agents involved are in stubbornness and no agreement is found yet;
- *agreement* when all the agents or an acceptable part of them, i.e. α agents where α is the degree of sharing, agree about a common angle.

In all the other cases, the MN continues. The system transition rules for 1-n MN are in Table 6. The following example shows a simple 1-n MN ending negatively or positively depending on the sharing degree decided in front of the beginning of the MN.

Example 5 Let Alice, Bob and Charles be three agents negotiating the definition of the term “vehicle”. Suppose that the initial viewpoint of Alice is

$$flex_A^0 = has3wheels \wedge hasSteeringWheel \wedge hasMotor$$

and her stubbornness knowledge is

$$stub_A = (has3wheels \vee has4wheels) \wedge hasSteeringWheel \\ \wedge (hasMotor \vee has2bicyclePedals \vee has4bicyclePedals)$$

Suppose that Bob's initial viewpoint is

$$flex_B^0 = has2wheels \wedge hasHandlebar \wedge has2bicyclePedals$$

and that his stubbornness knowledge is

$$stub_B = (has2wheels \vee has4wheels) \wedge (hasSteeringWheel \vee hasHandlebar) \wedge \\ (hasMotor \vee has2bicyclePedals \vee has4bicyclePedals)$$

and that the initial viewpoint of Charles is

$$flex_C^0 = has4wheels \wedge hasHandlebar \wedge hasTowBar$$

and his stubbornness knowledge is

$$\begin{aligned} stub_C = & has4wheels \wedge (hasHandlebar \vee hasSteeringWheel) \wedge \\ & (hasTowBar \vee hasMotor) \end{aligned}$$

Moreover, suppose that the MN is considered positive iff all the agents agree with a common angle, i.e there are $\alpha = 3$ agreeing agents. Alice is the first bidding agent, and thus she is the referee, and she proposes $flex_A^0$ to Bob and Charles. Bob and Charles receive the proposal and evaluate it. Bob tests that they are in essence disagreement because $(stub_B \vee \tau_{B,A}(flex_A^0)) \wedge \neg(flex_B^0 \wedge \tau_{B,A}(flex_A^0))$. Bob chooses the new CAF by a changing theory action (C) in

$$\begin{aligned} flex_B^1 = & (has2wheels \vee has4wheels) \wedge \\ & hasSteeringWheel \wedge (has2bicyclePedals \vee hasMotor) \end{aligned}$$

Bob uses the (ED) rule and sends his CAF to Alice:

$$\frac{A : flex_A^0 \quad (stub_B \vee \tau_{B,A}(flex_A^0)) \wedge \neg(flex_B^0 \wedge \tau_{B,A}(flex_A^0))}{B : \mathbf{essDis}(A : flex_A^0) \cap B : flex_B^1} \quad (ED)$$

Charles evaluates $flex_A^0$ and tests that they are in essence disagreement because $(stub_C \vee \tau_{C,A}(flex_A^0)) \wedge \neg(flex_C^0 \wedge \tau_{C,A}(flex_A^0))$. Charles chooses the new CAF by a changing theory action (C) in

$$flex_C^1 = has4wheels \wedge hasSteeringWheel \wedge hasTowBar$$

Charles uses the (ED) rule and sends his CAF to Alice:

$$\frac{A : flex_A^0 \quad (stub_C \vee \tau_{C,A}(flex_A^0)) \wedge \neg(flex_C^0 \wedge \tau_{C,A}(flex_A^0))}{C : \mathbf{essDis}(A : flex_A^0) \cap C : flex_C^1} \quad (ED)$$

The system continues the MN by:

$$\frac{*(A, B, C) \quad A : flex_A^0 \quad B : \mathbf{essDis}(A : flex_A^0) \quad C : \mathbf{essDis}(A : flex_A^0) \quad |\{A\}| \leq \alpha}{Negotiate(A, B, C)} \quad (NN)$$

Alice receives $flex_B^1$ and $flex_C^1$, and she has to make a weakening or a changing theory action because Bob and Charles did not say they were in agreement nor in relative disagreement. Alice performs a changing theory action by the rule (C) and her CAF is

$$flex_A^1 = has4wheels \wedge hasSteeringWheel \wedge (hasMotor \vee has2bicyclePedals)$$

Alice thinks she is in relative disagreement relation with Bob since $(flex_A^1 \rightarrow \tau_{A,B}(flex_B^1)) \wedge \neg(\tau_{A,B}(flex_B^1) \rightarrow flex_A^1)$.

Alice thinks she is in compatibility relation with Charles since $(flex_A^1 \vee \tau_{A,C}(flex_C^1)) \wedge (flex_A^1 \rightarrow \tau_{A,C}(flex_C^1)) \wedge \neg(\tau_{A,C}(flex_C^1) \rightarrow flex_A^1)$.

Alice uses the rule ($ED-RD$) to inform Bob they are in relative disagreement and ($ED-Co$) to inform Charles that they are in compatibility:

$$\frac{B : \mathbf{essDis}(A : flex_A^0) \cap B : flex_B^1 \quad (flex_A^1 \rightarrow \tau_{A,B}(flex_B^1)) \wedge \neg(\tau_{A,B}(flex_B^1) \rightarrow flex_A^1)}{A : \mathbf{relDis}(B : flex_B^1) \cap A : flex_A^1} \quad (ED-RD)$$

and

$$\frac{C : \mathbf{comp}(A : flex_A^0) \cap C : stub_C \quad (flex_A^1 \vee \tau_{A,C}(flex_C^1)) \wedge \neg(flex_A^1 \rightarrow \tau_{A,C}(flex_C^1)) \wedge \neg(\tau_{A,C}(flex_C^1) \rightarrow flex_A^1)}{A : \mathbf{essDis}(C : flex_C^1) \cap A : flex_A^1} \quad (ED-Co)$$

Bob receives $flex_A^1$ and he is in agreement with Alice by $(RD-Ag)$:

$$\frac{A : \mathbf{relDis}(B : flex_B^1) \cap A : flex_A^1}{B : \mathbf{agree}(A : flex_A^1) \cap B : \tau_{B,A}(flex_A^1)} \quad (RD-Ag)$$

Charles receives the proposal and evaluates it. Charles chooses the new CAF by a weakening action (C rule) in

$$flex_C^2 = has4wheels \wedge (hasHandlebar \vee hasSteeringWheel) \wedge (hasMotor \vee hasTowbar)$$

and $flex_C^2 = stub_C$. Charles tests that he is in compatibility relation with Alice since $(stub_C \vee \tau_{C,A}(flex_A^1)) \wedge \neg(stub_C \rightarrow \tau_{C,A}(flex_A^1)) \wedge \neg(\tau_{C,A}(flex_A^1) \rightarrow stub_C)$ and uses the $(Co-Co)$ rule to send his CAF to Alice:

$$\frac{A : \mathbf{comp}(C : stub_C) \cap A : flex_A^1 \quad (stub_C \vee \tau_{C,A}(flex_A^1)) \wedge \neg(stub_C \rightarrow \tau_{C,A}(flex_A^1)) \wedge \neg(\tau_{C,A}(flex_A^1) \rightarrow stub_C)}{C : \mathbf{comp}(A : flex_A^1) \cap C : stub_C} \quad (Co-Co)$$

The system continues the MN by:

$$\frac{* (A, B, C) \quad A : flex_A^0 \quad B : \mathbf{agree}(A : flex_A^1) \quad C : \mathbf{comp}(A : flex_A^1) \quad |\{A, B\}| \leq \alpha}{Negotiate(A, B, C)} \quad (NN)$$

Alice receives $\tau_{B,A}(flex_A^1)$ and $stub_C$ from Bob and Charles respectively and she has to make a weakening or a changing theory action because Charles did not say he was in agreement nor in relative disagreement with her. Alice performs a changing theory action by the rule (C) and her CAF is

$$flex_A^2 = has4wheels \wedge hasSteeringWheel \wedge (has4bicyclePedals \vee hasMotor)$$

Alice thinks she is in compatibility relation with Bob and Charles since $(flex_A^2 \vee \tau_{A,B}(\tau_{B,A}(flex_A^1))) \wedge \neg(flex_A^2 \rightarrow \tau_{A,B}(\tau_{B,A}(flex_A^1))) \wedge \neg(\tau_{A,B}(\tau_{B,A}(flex_A^1)) \rightarrow flex_A^2)$ and $(flex_A^2 \vee \tau_{A,C}(stub_C)) \wedge \neg(flex_A^2 \rightarrow \tau_{A,C}(stub_C)) \wedge \neg(\tau_{A,C}(stub_C) \rightarrow flex_A^2)$.

Alice uses the rule $(Ag-Co)$ to inform Bob she is in compatibility with him:

$$\frac{B : \mathbf{agree}(A : flex_A^1) \cap B : \tau_{B,A}(flex_A^1) \quad (flex_A^2 \vee \tau_{A,B}(\tau_{B,A}(flex_A^1))) \wedge \neg(flex_A^2 \rightarrow \tau_{A,B}(\tau_{B,A}(flex_A^1))) \wedge \neg(\tau_{A,B}(\tau_{B,A}(flex_A^1)) \rightarrow flex_A^2)}{A : \mathbf{comp}(B : \tau_{B,A}(flex_A^1)) \cap A : \tau_{A,B}(\tau_{B,A}(flex_A^1))} \quad (Ag-Co)$$

and the rule $(Co-Co)$ to inform Charles she is in compatibility with him:

$$\frac{C : \mathbf{comp}(A : flex_A^1) \cap C : stub_C \quad (flex_A^2 \vee \tau_{A,C}(stub_C)) \wedge \neg(flex_A^2 \rightarrow \tau_{A,C}(stub_C)) \wedge \neg(\tau_{A,C}(stub_C) \rightarrow flex_A^2)}{A : \mathbf{comp}(C : stub_C) \cap A : flex_A^2} \quad (Co-Co)$$

Bob receives $flex_A^2$ and he makes a weakening action by the rule W and his CAF is:

$$flex_B^3 = (has2wheels \vee has4wheels) \wedge (hasSteeringWheel \vee hasHandlebar) \wedge (hasMotor \vee has2bicyclePedals \vee has4bicyclePedals)$$

and $flex_B^3 = stub_B$. Bob uses the rule (*Co-Co*) and sends $stub_B$ to Alice.

$$\frac{A : \mathbf{comp}(B : flex_B^2) \cap A : flex_A^2 \quad (stub_B \vee \tau_{B,A}(flex_A^2)) \wedge \neg(stub_B \rightarrow \tau_{B,A}(flex_A^2)) \wedge \neg(\tau_{B,A}(flex_A^2) \rightarrow stub_B)}{B : \mathbf{comp}(A : flex_A^2) \cap B : stub_B} \quad (Co-Co)$$

Charles receives the proposal and evaluates it. Charles is in stubbornness thus he applies the (*S*) rule and proposes $stub_C$ to Alice.

Charles uses the (*Co-Co*) rule and sends his CAF to Alice:

$$\frac{A : \mathbf{comp}(C : stub_C) \cap A : flex_A^2 \quad (stub_C \vee \tau_{C,A}(flex_A^2)) \wedge \neg(stub_C \rightarrow \tau_{C,A}(flex_A^2)) \wedge \neg(\tau_{C,A}(flex_A^2) \rightarrow stub_C)}{C : \mathbf{comp}(A : flex_A^2) \cap C : stub_C} \quad (Co-Co)$$

The system continues the MN by:

$$\frac{* (A, B, C) \quad A : flex_A^2 \quad B : \mathbf{comp}(A : flex_A^2) \quad C : \mathbf{comp}(A : flex_A^2) \quad |\{A\}| \leq \alpha}{Negotiate(A, B, C)} \quad (NN)$$

Alice receives $stub_B$ and $stub_C$ and she has to make a weakening or a changing theory action because Charles did not say he was in agreement nor in relative disagreement with her. Alice performs a weakening action by the rule (*W*) and her CAF is

$$flex_A^3 = (has3wheels \vee has4wheels) \wedge hasSteeringWheel \\ \wedge (hasMotor \vee has2bicyclePedals \vee has4bicyclePedals)$$

and $flex_A^3 = stub_A$.

Alice thinks she is in compatibility relation with Bob since $(stub_A \vee \tau_{A,B}(stub_B)) \wedge (stub_A \rightarrow \tau_{A,B}(stub_B)) \wedge \neg(\tau_{A,B}(stub_B) \rightarrow stub_A)$.

Moreover, Alice thinks she is in compatibility relation with Charles since $(stub_A \vee \tau_{A,C}(stub_C)) \wedge (stub_A \rightarrow \tau_{A,C}(stub_C)) \wedge \neg(\tau_{A,C}(stub_C) \rightarrow stub_A)$.

Alice uses the (*Co-Co*) rule to inform Bob and Charles that they are in compatibility relation:

$$\frac{B : \mathbf{comp}(A : flex_A^2) \cap B : stub_B \quad (stub_A \vee \tau_{A,B}(stub_B)) \wedge \neg(stub_A \rightarrow \tau_{A,B}(stub_B)) \wedge \neg(\tau_{A,B}(stub_B) \rightarrow stub_A)}{A : \mathbf{comp}(B : stub_B) \cap A : stub_A} \quad (Co-Co)$$

and

$$\frac{C : \mathbf{comp}(A : flex_A^2) \cap C : stub_C \quad (stub_A \vee \tau_{A,C}(stub_C)) \wedge \neg(stub_A \rightarrow \tau_{A,C}(stub_C)) \wedge \neg(\tau_{A,C}(stub_C) \rightarrow stub_A)}{A : \mathbf{comp}(C : stub_C) \cap A : stub_A} \quad (Co-Co)$$

Bob and Charles receive the proposal and evaluate it. They are in stubbornness thus they apply the (*S*) rule and they propose $stub_B$ and $stub_C$ to Alice respectively.

They use the (*Co-Co*) rule and send their CAFs to Alice:

$$\frac{A : \mathbf{comp}(B : stub_B) \cap A : stub_A \quad (stub_B \vee \tau_{B,A}(stub_A)) \wedge \neg(stub_B \rightarrow \tau_{B,A}(stub_A)) \wedge \neg(\tau_{B,A}(stub_A) \rightarrow stub_B)}{B : \mathbf{comp}(A : stub_A) \cap B : stub_B} \quad (Co-Co)$$

and

$$\frac{A : \mathbf{comp}(C : stub_C) \cap A : stub_A \quad (stub_C \vee \tau_{C,A}(stub_A)) \wedge \neg(stub_C \rightarrow \tau_{C,A}(stub_A)) \wedge \neg(\tau_{C,A}(stub_A) \rightarrow stub_C)}{C : \mathbf{comp}(A : stub_A) \cap C : stub_C} \quad (Co-Co)$$

The system closes the MN by:

$$\frac{* (A, B, C) \quad A : stub_A \quad B : \mathbf{comp}(A : stub_A) \quad C : \mathbf{agree}(A : stub_A) \quad |\{A\}| \leq \alpha \quad \text{for all } i \in \mathbf{Ag}.i : \psi \text{ and } stub_i \leftrightarrow \psi}{Disagreement(A, B, C)} \quad (DD)$$

with a negative outcome. The negotiation would be positively ending by rule (AA) whether the sharing degree were $\alpha = 2$ when Alice proposes $A : flex_A^1$ to Bob and Charles. \square

As for the rules for bilateral MN, we can show the consistency and adequateness of the rules for 1- n MN.

Theorem 3 *MND for 1- n MN is consistent.*

Proof Consider n agents represented in the *MND* system with sets L_{S_1}, \dots, L_{S_n} of stubbornness formulas and sets L_{F_1}, \dots, L_{F_n} of flexible formulas. The proof of the consistency of *MND* for 1- n MN is similar to the proof of Theorem 1. We show that if a Σ_i formula ξ is inferred using the *MND* 1- n rules, or, in other terms, is deduced as a theorem in the system, then ξ represents a proposal that is acceptable by all or at least α agents. In other words, we aim at proving that when the rules yield ξ then ξ generalizes at least α languages among L_{F_1}, \dots, L_{F_n} and is generalized by at least α languages among L_{S_1}, \dots, L_{S_n} . Let us now consider a formula ξ that is acceptable by at least α agents, and let us consider the rules that produce transitions in the system. In particular, if ξ is inferred by means of one of the rules (AD), (ED), (I), (Co), (Ag) for the second proposing agent, or by means of one of the rules given in Fig. 3 or in Fig. 5 for the following proposing agent and the auctioneer, then the possible results of the step described above are given by the application of the system transition rules. Evidently, if ξ is inferred, then the rule (DD) does not apply. If (NN) applies, and one more inference is performed, then the rules (W), (C), (S) allow us to infer a different formula. Suppose now, by contradiction, that the new formula ξ is not acceptable by more than $n - \alpha$ agents (in the sense that either it is not a generalization of their set of flexible formulas or it is not generalized by their sets of stubbornness formulas). As a consequence, these agents have called themselves away, as we stated above. This, however, is impossible, by construction of the rules for the second and following proposals, and for the auctioneer. Conversely, if the transition rule (DD) applies and, therefore, more than $n - \alpha$ agents have incompatible viewpoints, then ξ is not inferred through the system, because it is not a generalization of the flexible sets of formulas and generalizes by the stubbornness sets of formulas of at least α agents. Clearly, by means of the full set of rules, it is not possible to do so when the agents have consistent viewpoints. \square

We say that a deductive system is adequate to represent a MN between n agents when it infers an outcome iff an agreement is reachable among the agents otherwise it does not produce any result.

Definition 7 A deductive system \mathcal{R} is *adequate* to represent the MN process among n agents when

$$\mathcal{R} \text{ infers } \begin{cases} \varphi & \text{iff there exists } \mathbf{Ag}' \subseteq \mathbf{Ag} \text{ s.t. } |\mathbf{Ag}'| = m \geq \alpha \text{ and for all } i \in \mathbf{Ag}' \\ & \text{there exists } k \in \mathbb{N} \text{ s.t. } \mathcal{A}_i^k \models (flex_i^k \rightarrow \varphi) \wedge (\varphi \rightarrow stub_i) \\ \perp & \text{otherwise} \end{cases}$$

where Ag is the set of the agents with $|Ag| = n$, $\varphi \in \bigcup_{i \in Ag} L_i$ and α is the minimum number of agreeing agents to consider positive the outcome of the MN.

Theorem 4 *MND is adequate to represent the MN of n agents.*

Proof We consider n agents of which m agents have consistent viewpoints where $n \geq m \geq \alpha$, namely such that there exists a possible common angle. Their stubbornness sets and their flexible sets of formulas are pairwise in one of the EGG/YOLK configurations except number 1. Suppose now that the *MND* system infers a Σ_i formula ξ . Then, ξ is a common angle for at least α agents. Conversely, suppose that *MND* does not infer any Σ_i formula. Then, some of the m agents are in call-away situation with the received proposal. Suppose now that there are more than $n - \alpha$ agents having pairwise inconsistent viewpoints (configuration 1). The relation established is absolute disagreement. The result is that no formula can be inferred through the system, which is consistent by Theorem 3. Hence, overall, the system is adequate. \square

3.3 MN Process Development

In this section, we show how the MN process develops in terms of the changing of the relations between the EGG/YOLKs of the agents. We model the multiparty MN among n agents as an English Auction Game in which the auctioneer negotiates simultaneously with $n - 1$ agents so that it can be considered as $n - 1$ bilateral MNs. For this reason we show here how the relation of the EGG/YOLKs changes during a MN process only for bilateral MN. As said above, the stubborn knowledge of the agents never changes during the negotiation but only the flexible part may differ from one step to the next one of a MN. The evolution of the relations between the EGG/YOLKs is different when the stubborn knowledge of the agents are inconsistent, or in a generalization relation, or just consistent or equivalent. We show below the MN development in all the cases.

Agents in MN make offers *flex* such that for each agent i :

$$flex_i = stub_i \wedge \varphi$$

where $stub_i$ is the stubbornness knowledge formula and φ is the flexible part of $flex_i$. Whenever an agent receive the opponent proposal, she does not know which is its flexible part and which the stubbornness one. She only knows which is the relation of the received proposal with respect to her own stubbornness and flexible knowledge.

Only the supervisor system knows the stubbornness knowledge of all the agents. As said above, the stubbornness knowledge is unquestionable and it never changes during the negotiation. Therefore, the MN process can be represented as a path of a graph in which nodes are the EGG/YOLK configurations and edges are the result of the usage of a bidding rule (Table 2 and Table 3).

Suppose that the agents have inconsistent stubbornness knowledge; whatever the deductive rules they use, they remain related as in configuration number 1 and the knowledge is described by a logical formula involving the stubbornness formulas of the agents (see Table 7).

In the following subsections, we describe all the MN situations with respect to the relations of the stubbornness knowledge of the agents.

3.3.1 Equivalent Stubbornness Knowledge

Suppose that the agents have equivalent stubbornness sets. Then

$$stub_i \leftrightarrow stub_j$$

The relations between two sets are defined by means of the intersection of their interiors and their exteriors. Given two sets, there are, in theory, two relations between their interiors (either the set intersection is empty or not), two between their exteriors and two relations between the interior of one of them and the exterior of the other one. The possible configurations are then eight, but some of them are absurd (for instance when the intersection of the interiors is empty, the intersection of the exteriors cannot be empty, and vice versa). This presentation issues have been studied deeply in the past and summarised in the spatial reasoning framework known as the Region Connection Calculus (RCC-5). This calculus provides five relations for the cases in which the sets coincide (EQ), two order relations of proper part (PP and PP^{-1}), the relation of proper overlapping (PO) and the relation of disjointness (DR). The equivalence relation between stubbornness theories relations (RCC5) by EQ.

In Table 8, we show the possible yolk configurations and we give a statement representing the configuration, i.e. the negotiation state.


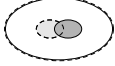
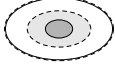
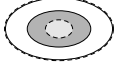

Configuration	Formula
42a 	$(stub_i \leftrightarrow stub_j) \wedge \neg(flex_i \wedge flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
42b 	$(stub_i \leftrightarrow stub_j) \wedge (flex_i \vee flex_j) \wedge \neg(flex_i \rightarrow flex_j) \wedge \neg(flex_j \rightarrow flex_i) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
42c 	$(stub_i \leftrightarrow stub_j) \wedge (flex_i \rightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
42d 	$(stub_i \leftrightarrow stub_j) \wedge (flex_j \rightarrow flex_i) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
42e 	$(stub_i \leftrightarrow stub_j) \wedge (flex_i \leftrightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$

Table 8: Configurations for equivalent stubbornness sets. Agent i is identified by plain lines and agent j by dashed lines.

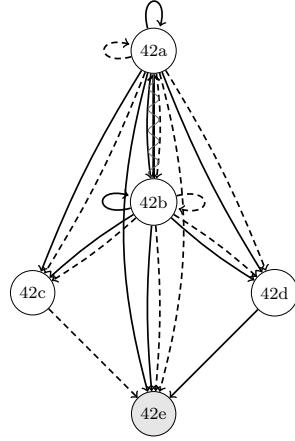


Fig. 9 Transition graph for equivalent stubbornness knowledge. Nodes are coloured: the gray node is the configuration of the positive outcome of the negotiation process.

Figure 9 depicts the graph of the possible negotiation relations of the agents during the negotiation. The nodes are the EGG/YOLK configurations and the edges are coloured by the agent who makes the next bid. The gray node identifies the positive outcome of the negotiation.

All the rules the agents use when their stubbornness knowledge are equivalent are *legitimate*. A rule is legitimate when it can be used in a specific configuration. In the following example, we show how deductive rules of MND are used and their effects in the EGG/YOLK configurations when the stubbornness knowledge of agents are equivalent.

Example 6 Suppose Alice and Bob are related as in configuration 42a. Alice, A , is the first bidding agent and she proposes $flex_A^0$ to Bob, B . Bob receives the proposal and evaluates it. Bob tests that they are in essence disagreement and generalizes his initial viewpoint $flex_B^0$ by:

$$\frac{flex_B^0 \rightarrow flex_B^1 \quad \neg(stub_B \leftrightarrow flex_B^0)}{flex_B^1} (W)$$

and he checks the provisional negotiation situation by:

$$\frac{A : flex_A^0 \quad \neg(flex_B^0 \wedge \tau_{B,A}(flex_A^0)) \wedge (stub_B \vee \tau_{B,A}(flex_A^0))}{B : \mathbf{essDis}(A : flex_A^0) \cap B : flex_B^1} (ED)$$

Bob says to Alice that they are in essence disagreement and makes a proposal $flex_B^1$.

The system continues the MN by:

$$\frac{*(A, B) \quad A : flex_A^0 \quad B : \mathbf{essDis}(A : flex_A^0) \quad B : flex_B^1}{Negotiate(A, B)} (N)$$

Alice receives $flex_B^1$ and she makes a weakening or a changing action because Bob said they are not in agreement nor in relative disagreement. Suppose Alice generalizes

her CAF by:

$$\frac{flex_A^0 \rightarrow flex_A^1 \quad \neg(stub_A \leftrightarrow flex_A^0)}{flex_A^1} (W)$$

Alice tests the negotiation relation by:

$$\frac{B : \mathbf{essDis}(A : flex_A^0) \cap B : flex_B^1 \quad (flex_A^1 \vee \tau_{A,B}(flex_B^1)) \wedge \neg(flex_A^1 \rightarrow \tau_{A,B}(flex_B^1)) \wedge \neg(\tau_{A,B}(flex_B^1) \rightarrow flex_A^1)}{A : \mathbf{comp}(B : flex_B^1) \cap A : flex_A^1} (ED-Co)$$

Alice says to Bob that they are in compatibility and makes a proposal $flex_A^1$.

The system continues the MN by:

$$\frac{*(B, A) \quad B : flex_B^1 \quad A : \mathbf{comp}(B : flex_B^1) \quad A : flex_A^1}{Negotiate(B, A)} (N)$$

Bob receives $flex_A^1$ and he makes a weakening or a changing action because Alice said they are not in agreement nor in relative disagreement. Suppose Bob changes his CAF by:

$$\frac{flex_B^1 \quad \neg(stub_B \leftrightarrow flex_B^1) \quad \neg(flex_B^1 \rightarrow flex_B^2) \quad \neg(flex_B^2 \rightarrow flex_B^1)}{flex_B^2} (C)$$

Bob tests the negotiation relation by:

$$\frac{A : \mathbf{comp}(B : flex_B^1) \cap A : flex_A^1 \quad (flex_A^2 \vee \tau_{B,A}(flex_A^1)) \wedge \neg(flex_A^2 \rightarrow \tau_{B,A}(flex_A^1)) \wedge \neg(flex_A^2 \leftarrow \tau_{B,A}(flex_A^1))}{B : \mathbf{comp}(A : flex_A^1) \cap B : flex_B^2} (Co-Co)$$

Bob says to Alice that they are in compatibility and makes a proposal $flex_B^2$.

The system continues the MN by:

$$\frac{*(A, B) \quad A : flex_A^1 \quad B : \mathbf{comp}(A : flex_A^1) \quad B : flex_B^2}{Negotiate(A, B)} (N)$$

Alice receives $flex_B^2$ and she makes a weakening or a changing action because Bob said they are not in agreement nor in relative disagreement. Suppose Alice changes her CAF by:

$$\frac{flex_A^2 \quad \neg(stub_A \leftrightarrow flex_A^2) \quad \neg(flex_A^2 \rightarrow flex_A^3) \quad \neg(flex_A^3 \rightarrow flex_A^2)}{flex_A^3} (C)$$

Alice tests the negotiation relation by:

$$\frac{B : \mathbf{comp}(A : flex_A^2) \cap B : flex_B^2 \quad (flex_A^3 \rightarrow \tau_{A,B}(flex_B^3))}{A : \mathbf{agree}(B : flex_A^2) \cap A : \tau_{A,B}(flex_B^2)} (Co-Ag)$$

Alice says to Bob that they are in agreement and that they have a common angle that is $flex_B^2$.

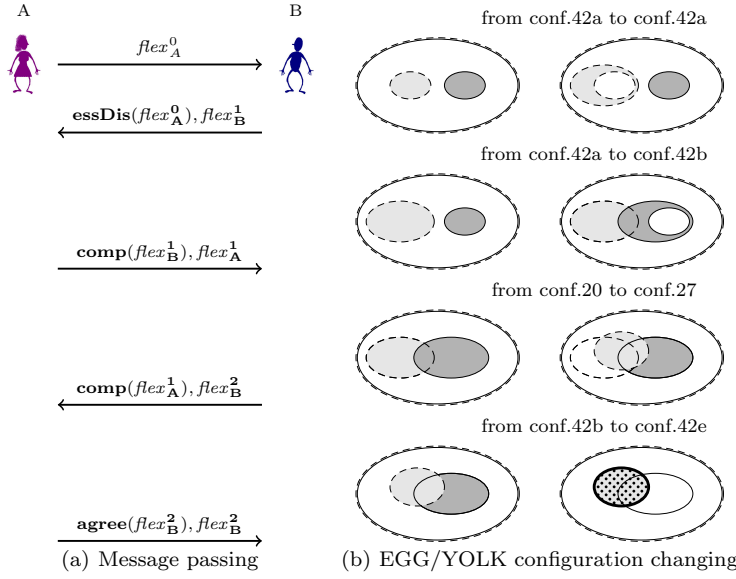


Fig. 10 A MN scenario between Alice and Bob with equivalent stubbornness knowledge: the message passing flow (a) and the changes of their CAFs (b). Alice is identified by plain lines and Bob by dashed lines. White yolks represent the precedent proposal of the agent and the dotted yolk is the positive outcome of the scenario.

The system closes the MN by:

$$\frac{* (B, A) \quad B : flex_B^2 \quad A : \mathbf{agree}(B : flex_B^2)}{Agreement(B, A)} (A)$$

with a positive outcome, $flex_B^2$.

In Figure 10, we show the message passing flow between Alice and Bob and the changes of the EGG/YOLK configurations. The MN results in a path, shown in Figure 11, from node 8 to node 41 of the graph in Figure 9.

□

3.3.2 Generalized Stubbornness Knowledge

Suppose that one agent's stubbornness set is a *generalization* of the theory of the opponent, i.e. they are consistent and one is a restriction of the other. Then

$$stub_i \rightarrow stub_j$$

The generalization (weakening, relaxing, etc.) relation between stubbornness theories is represented in RCC5 as the partial proper part relation between eggs. We assumed that the stubbornness part of the agent theory never changes, then the models satisfying it are fixed at the beginning of the negotiation process.

On the other hand, the flexible sets are relaxed or changed during the negotiation process so that the models satisfying them change during the negotiation. The *flexible models* are the yolks of the RCC theory.

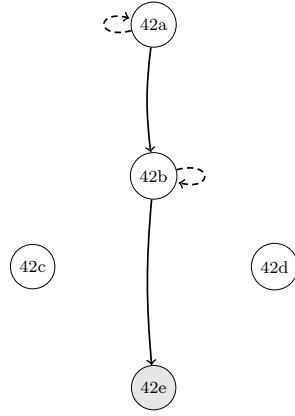
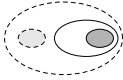
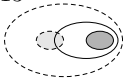
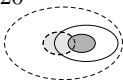
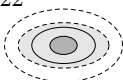
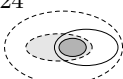
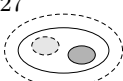


Fig. 11 The MN path of the Alice and Bob message passing in Figure 10.

In Table 9, we show the possible yolk configurations and we give a statement representing the configuration, i.e. the negotiation state.

Configuration	Formula
8 	$(stub_i \rightarrow stub_j) \wedge \neg(flex_i \wedge flex_j) \wedge (flex_i \rightarrow stub_j) \wedge \neg(flex_j \wedge stub_i)$
13 	$(stub_i \rightarrow stub_j) \wedge \neg(flex_i \wedge flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \vee stub_i) \wedge \neg(flex_j \rightarrow stub_i) \wedge \neg(stub_i \rightarrow flex_j)$
20 	$(stub_i \rightarrow stub_j) \wedge (flex_i \vee flex_j) \wedge \neg(flex_i \rightarrow flex_j) \wedge \neg(flex_i \rightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \vee stub_i) \wedge \neg(flex_j \rightarrow stub_i) \wedge \neg(stub_i \rightarrow flex_j)$
22 	$(stub_i \rightarrow stub_j) \wedge (flex_i \rightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (stub_i \rightarrow flex_j)$
24 	$(stub_i \rightarrow stub_j) \wedge (flex_i \rightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \vee stub_i) \wedge \neg(flex_j \rightarrow stub_i) \wedge \neg(stub_i \rightarrow flex_j)$
27 	$(stub_i \rightarrow stub_j) \wedge \neg(flex_i \wedge flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$

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Table 9 – continued from previous page

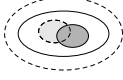
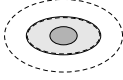
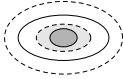
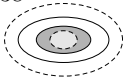
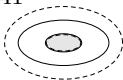
Configuration	Formula
32 	$(stub_i \rightarrow stub_j) \wedge (flex_i \vee flex_j) \wedge \neg(flex_i \rightarrow flex_j) \wedge \neg(flex_j \rightarrow flex_i) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
34 	$(stub_i \rightarrow stub_j) \wedge (flex_i \rightarrow flex_j) \wedge (flex_j \leftrightarrow stub_i) \wedge (flex_i \rightarrow stub_j)$
37 	$(stub_i \rightarrow stub_j) \wedge (flex_i \rightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
38 	$(stub_i \rightarrow stub_j) \wedge (flex_j \rightarrow flex_i) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
41 	$(stub_i \rightarrow stub_j) \wedge (flex_i \leftrightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$

Table 9: Configurations for generalized/restricted stubbornness sets. The stubbornness knowledge of agent i , identified by plain lines, is generalized by the stubbornness knowledge of the agent j , identified by dashed lines.

Figure 12 depicts the graph of the possible negotiation relations of the agents during the negotiation. The nodes are the EGG/YOLK configurations and the edges are colored by the agent who makes the next bid. The gray node identifies the positive outcome of the negotiation.

All the rules of agent i , identified by plain lines, are legitimate. The violation rules are used only by the agent j , identified by dashed lines.

In the following example, we show how deductive rules of MND are used and their effects in the EGG/YOLK configurations when the stubbornness knowledge of agents are in a generalization/restriction relation.

Example 7 Suppose Alice and Bob are related as in configuration 8. Alice, A , is the first bidding agent and she proposes $flex_A^0$ to Bob, B . Bob receives the proposal and evaluates it. Bob tests that they are in essence disagreement. Bob generalizes his initial viewpoint $flex_B^0$ by:

$$\frac{flex_B^0 \rightarrow flex_B^1 \quad \neg(stub_B \leftrightarrow flex_B^0)}{flex_B^1} (W)$$

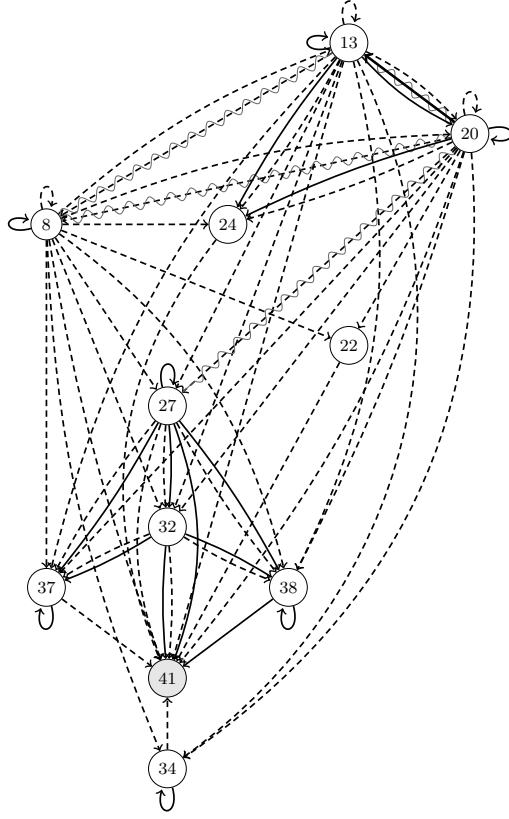


Fig. 12 Transition graph for generalized (or restricted) stubbornness knowledge: the stubbornness knowledge of agent i , identified by plain lines, is a restriction of the stubbornness knowledge of agent j , identified by dashed lines. The nodes are colored: the gray node is the configuration of the positive outcome of the negotiation process.

and he checks the provisional negotiation situation by:

$$\frac{A : flex_A^0 \quad \neg(flex_B^0 \wedge \tau_{B,A}(flex_A^0)) \wedge (stub_B \vee \tau_{B,A}(flex_A^0))}{B : \mathbf{essDis}(A : flex_A^0) \cap B : flex_B^1} (ED)$$

Bob says to Alice that they are in essence disagreement and makes a proposal $flex_B^1$.

The system continues the MN by:

$$\frac{*(A, B) \quad A : flex_A^0 \quad B : \mathbf{essDis}(A : flex_A^0) \quad B : flex_B^1}{Negotiate(A, B)} (N)$$

Alice receives $flex_B^1$ and she makes a weakening or a changing action because Bob said they are not in agreement nor in relative disagreement. Suppose Alice changes her CAF by:

$$\frac{flex_A^0 \quad \neg(stub_A \leftrightarrow flex_A^0) \quad \neg(flex_A^0 \rightarrow flex_A^1) \quad \neg(flex_A^1 \rightarrow flex_A^0)}{flex_A^1} (C)$$

Alice tests the negotiation relation by:

$$\frac{B : \mathbf{essDis}(A : flex_A^0) \cap B : flex_B^1 \quad (flex_A^1 \vee \tau_{A,B}(flex_B^1)) \wedge \neg(flex_A^1 \rightarrow \tau_{A,B}(flex_B^1)) \wedge \neg(flex_A^1 \leftarrow \tau_{A,B}(flex_B^1))}{A : \mathbf{comp}(B : flex_B^1) \cap A : flex_A^1} \quad (ED-Co)$$

Alice says to Bob that they are in compatibility and makes a proposal $flex_A^1$.

The system continues the MN by:

$$\frac{*(B, A) \quad B : flex_B^1 \quad A : \mathbf{comp}(B : flex_B^1) \quad A : flex_A^1}{Negotiate(B, A)} \quad (N)$$

Bob receives $flex_A^1$ and he makes a weakening or a changing action because Alice said they are not in agreement nor in relative disagreement. Suppose Bob changes his CAF by:

$$\frac{flex_B^1 \quad \neg(stub_B \leftrightarrow flex_B^1) \quad \neg(flex_B^1 \rightarrow flex_B^2) \quad \neg(flex_B^2 \rightarrow flex_B^1)}{flex_B^2} \quad (C)$$

Bob tests the negotiation relation and makes a violation by:

$$\frac{A : \mathbf{comp}(B : flex_B^1) \cap A : flex_A^1 \quad (stub_B \vee \tau_{B,A}(flex_A^1)) \wedge \neg(flex_B^2 \wedge \tau_{B,A}(flex_A^1))}{B : \mathbf{essDis}(A : flex_A^1) \cap B : flex_B^2} \quad (Co-ED)$$

Bob says to Alice that they are in essence disagreement and makes a proposal $flex_B^2$.

The system continues the MN by:

$$\frac{*(A, B) \quad A : flex_A^1 \quad B : \mathbf{essDis}(A : flex_A^1) \quad B : flex_B^2}{Negotiate(A, B)} \quad (N)$$

Alice receives $flex_B^2$ and she makes a weakening or a changing action because Bob said they are not in agreement nor in relative disagreement. Suppose Alice changes her CAF by:

$$\frac{flex_A^1 \quad \neg(stub_A \leftrightarrow flex_A^1) \quad \neg(flex_A^1 \rightarrow flex_A^2) \quad \neg(flex_A^2 \rightarrow flex_A^1)}{flex_A^2} \quad (C)$$

Alice tests the negotiation relation by:

$$\frac{B : \mathbf{essDis}(A : flex_A^1) \cap B : flex_B^2 \quad (flex_A^2 \vee \tau_{A,B}(flex_B^2)) \wedge \neg(flex_A^2 \rightarrow \tau_{A,B}(flex_B^2)) \wedge \neg(flex_A^2 \leftarrow \tau_{A,B}(flex_B^2))}{A : \mathbf{comp}(B : flex_B^2) \cap A : flex_A^2} \quad (ED-Co)$$

Alice says to Bob that they are in compatibility and makes a proposal $flex_A^2$.

$$\frac{*(B, A) \quad B : flex_B^2 \quad A : \mathbf{comp}(B : flex_B^2) \quad A : flex_A^2}{Negotiate(B, A)} \quad (N)$$

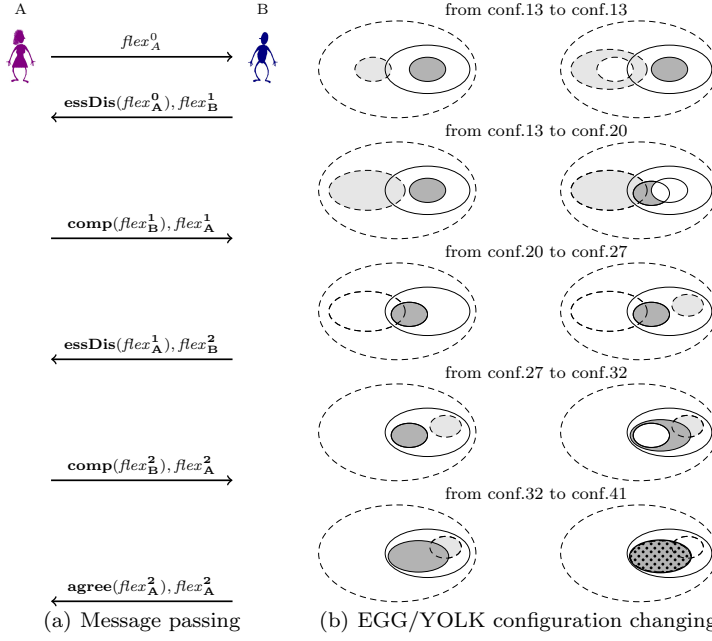


Fig. 13 A MN scenario between Alice and Bob with stubbornness knowledge of Alice is a restriction of the stubbornness knowledge of Bob: the message passing flow (a) and the changes of their CAFs (b). White yolks represent the precedent proposal of the agent and the dotted gray yolk is the positive outcome of the scenario.

Bob receives $flex_A^2$ and he makes a weakening or a changing action because Alice said they are not in agreement nor in relative disagreement. Suppose Bob changes his CAF by:

$$\frac{flex_B^2 \quad \neg(stub_B \leftrightarrow flex_B^2) \quad \neg(flex_B^2 \rightarrow flex_B^3) \quad \neg(flex_B^3 \rightarrow flex_B^2)}{flex_B^3} (C)$$

Bob tests the negotiation relation by:

$$\frac{A : \mathbf{comp}(B : flex_B^2) \cap A : flex_A^2 \quad (flex_B^3 \rightarrow \tau_{B,A}(flex_A^2))}{B : \mathbf{agree}(A : flex_A^2) \cap B : \tau_{B,A}(flex_A^2)} (Co-Ag)$$

Bob says to Alice that they are in agreement and that they have a common angle that is $flex_A^2$.

The system closes the MN by:

$$\frac{*(A, B) \quad A : flex_A^2 \quad B : \mathbf{agree}(A : flex_A^2)}{Agreement(A, B)} (A)$$

with a positive outcome, $flex_A^2$.

In Figure 13 we show the message passing flow between Alice and Bob and the changes of the EGG/YOLK configurations.

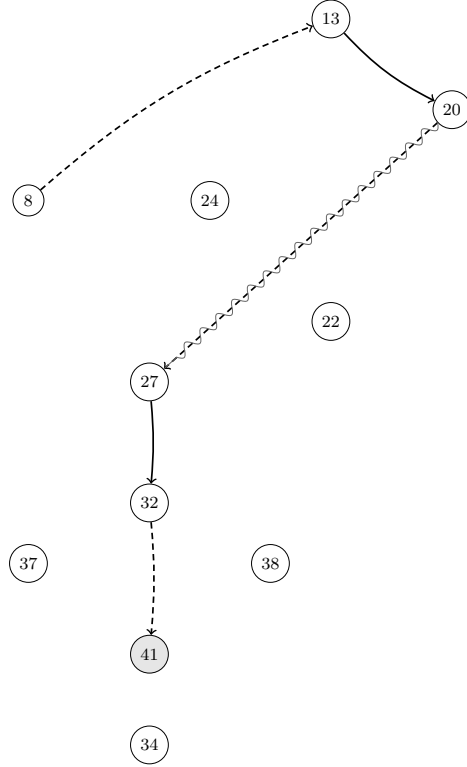


Fig. 14 The MN path of the Alice and Bob message passing in Figure 13.

The MN results in a path, showed in Figure 14, from node 8 to node 41 of the graph in Figure 12.

□

3.3.3 Consistent Stubbornness Knowledge

Suppose that the agents' stubbornness knowledge are *compatible*, i.e. they are consistent and no one is a restriction or a generalization of the other. Then

$$(stub_i \vee stub_j) \wedge \neg(stub_i \rightarrow stub_j) \wedge \neg(stub_j \rightarrow stub_i)$$

The compatibility relation between stubbornness theories is represented in RCC5 as the partial overlapping relation between eggs. We assumed that if the stubbornness part of the agent theory never changes, then the models satisfying it are fixed at the beginning of the negotiation process.

On the other hand, the flexible sets are relaxed or changed during the negotiation process so the models satisfying them change during the negotiation. The *flexible models* are the yolks of the RCC theory.

In Table 10, we show the possible yolk configurations and we give a statement representing the configuration, i.e. the negotiation state.

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Table 10 – continued from previous page


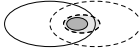





Configuration	Formula
17 	$(stub_i \vee stub_j) \wedge \neg(stub_i \rightarrow stub_j) \wedge \neg(stub_j \rightarrow stub_i) \wedge (flex_j \rightarrow flex_i) \wedge (flex_i \vee stub_j) \wedge \neg(flex_i \rightarrow stub_j) \wedge \neg(stub_j \rightarrow flex_i) \wedge (flex_j \rightarrow stub_i)$
18 	$(stub_i \vee stub_j) \wedge \neg(stub_i \rightarrow stub_j) \wedge \neg(stub_j \rightarrow stub_i) \wedge (flex_i \rightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \vee stub_i) \wedge \neg(flex_j \rightarrow stub_i) \wedge \neg(stub_i \rightarrow flex_j)$
25 	$(stub_i \vee stub_j) \wedge \neg(stub_i \rightarrow stub_j) \wedge \neg(stub_j \rightarrow stub_i) \wedge \neg(flex_i \wedge flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
28 	$(stub_i \vee stub_j) \wedge \neg(stub_i \rightarrow stub_j) \wedge \neg(stub_j \rightarrow stub_i) \wedge (flex_i \vee flex_j) \wedge \neg(flex_i \rightarrow flex_j) \wedge \neg(flex_j \rightarrow flex_i) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
29 	$(stub_i \vee stub_j) \wedge \neg(stub_i \rightarrow stub_j) \wedge \neg(stub_j \rightarrow stub_i) \wedge (flex_j \rightarrow flex_i) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
30 	$(stub_i \vee stub_j) \wedge \neg(stub_i \rightarrow stub_j) \wedge \neg(stub_j \rightarrow stub_i) \wedge (flex_i \rightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$
39 	$(stub_i \vee stub_j) \wedge \neg(stub_i \rightarrow stub_j) \wedge \neg(stub_j \rightarrow stub_i) \wedge (flex_i \leftrightarrow flex_j) \wedge (flex_i \rightarrow stub_j) \wedge (flex_j \rightarrow stub_i)$

Table 10: Configurations for consistent stubbornness sets. The stubbornness knowledge of the agent i , identified by plain lines, is only consistent by the stubbornness knowledge of the agent j , identified by dashed lines.

Figure 15 depicts the graph of the possible negotiation relations of the agents during the negotiation. The nodes are the EGG/YOLK configurations and the edges are colored by the agent who makes the next bid. The gray node identifies the positive outcome of the negotiation.

Both agents may make legitimate or violation actions, thus they may use or not the violation rules in Table 3.

In the following example, we show how deductive rules of MND are used and their effects in the EGG/YOLK configurations when the stubbornness knowledge of agents are consistent and no generalization or restriction relation exist between them.

Example 8 Suppose Alice and Bob are related as in configuration 2. Alice, A , is the first bidding agent and she proposes $flex_A^0$ to Bob, B . Bob receives the proposal and evaluates it. Bob tests that they are in absolute disagreement. Bob generalizes his

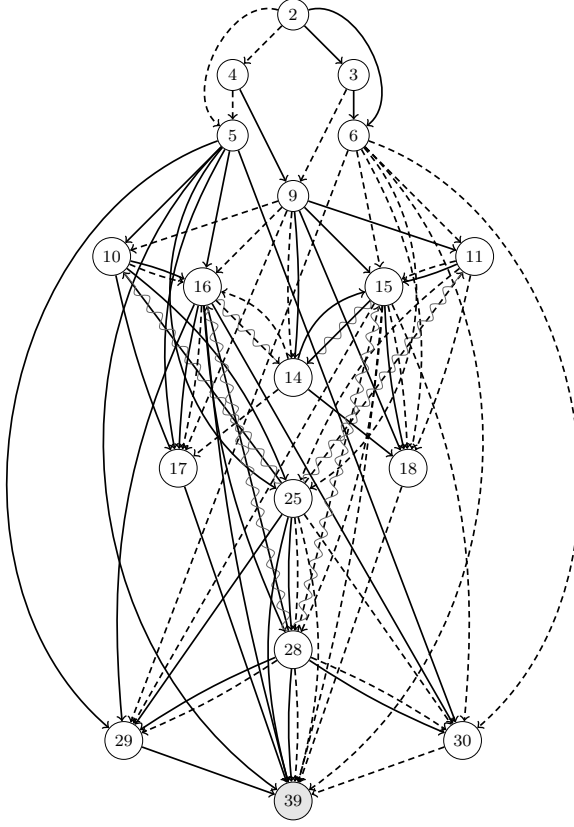


Fig. 15 Transition graph for consistent and not generalized/restricted stubbornness knowledge: the stubbornness knowledge of agent i , identified by plain lines, is not a restriction of the stubbornness knowledge of agent j , identified by dashed lines, and vice versa but they have shared semantical structures. The nodes are coloured: the gray node is the configuration of the positive outcome of the negotiation process.

initial viewpoint $flex_B^0$ by:

$$\frac{flex_B^0 \rightarrow flex_B^1 \quad \neg(stub_B \leftrightarrow flex_B^0)}{flex_B^1} (W)$$

and he checks the provisional negotiation situation by:

$$\frac{A : flex_A^0 \quad \neg(stub_B \wedge \tau_{B,A}(flex_A^0))}{B : \mathbf{absDis}(A : flex_A^0) \cap B : flex_B^1} (AD)$$

Bob says to Alice that they are in absolute disagreement and makes a proposal $flex_B^1$.

The system continues the MN by:

$$\frac{*(A, B) \quad A : flex_A^0 \quad B : \mathbf{absDis}(A : flex_A^0) \quad B : flex_B^1}{Negotiate(A, B)} (N)$$

Alice receives $flex_B^1$ and she makes a weakening or a changing action because Bob said they are not in agreement nor in relative disagreement. Suppose Alice changes her CAF by:

$$\frac{flex_A^0 \quad \neg(stub_A \leftrightarrow flex_A^0) \quad \neg(flex_A^0 \rightarrow flex_A^1) \quad \neg(flex_A^1 \rightarrow flex_A^0)}{flex_A^1} (C)$$

Alice tests the negotiation relation by:

$$\frac{B : \mathbf{absDis}(A : flex_A^0) \cap B : flex_B^1 \quad (stub_A \vee \tau_{A,B}(flex_B^1)) \wedge \neg(flex_A^1 \wedge \tau_{A,B}(flex_B^1))}{A : \mathbf{essDis}(B : flex_B^1) \cap A : flex_A^1} (AD-ED)$$

Alice says to Bob that they are in essence disagreement and makes a proposal $flex_A^1$.

The system continues the MN by:

$$\frac{*(B, A) \quad B : flex_B^1 \quad A : \mathbf{essDis}(B : flex_B^1) \quad A : flex_A^1}{Negotiate(B, A)} (N)$$

Bob receives $flex_A^1$ and he makes a weakening or a changing action because Alice said they are not in agreement nor in relative disagreement. Suppose Bob changes her CAF by:

$$\frac{flex_B^1 \quad \neg(stub_B \leftrightarrow flex_B^1) \quad \neg(flex_B^1 \rightarrow flex_B^2) \quad \neg(flex_B^2 \rightarrow flex_B^1)}{flex_B^2} (C)$$

Bob tests the negotiation relation by:

$$\frac{A : \mathbf{essDis}(B : flex_B^1) \cap A : flex_A^1 \quad (stub_B \vee \tau_{B,A}(flex_A^1)) \wedge \neg(flex_B^2 \wedge \tau_{B,A}(flex_A^1))}{B : \mathbf{essDis}(A : flex_A^1) \cap B : flex_B^2} (ED-ED)$$

Bob says to Alice that they are in essence disagreement and makes a proposal $flex_B^2$.

The system continues the MN by:

$$\frac{*(A, B) \quad A : flex_A^1 \quad B : \mathbf{essDis}(A : flex_A^1) \quad B : flex_B^2}{Negotiate(A, B)} (N)$$

Alice receives $flex_B^2$ and she makes a weakening or a changing action because Bob said they are not in agreement nor in relative disagreement. Suppose Alice generalizes her CAF by:

$$\frac{flex_A^1 \rightarrow flex_A^2 \quad \neg(stub_A \leftrightarrow flex_A^1)}{flex_A^2} (W)$$

Alice tests the negotiation relation by:

$$\frac{B : \mathbf{essDis}(A : flex_A^1) \cap B : flex_B^2 \quad (flex_A^2 \vee \tau_{A,B}(flex_B^2)) \wedge \neg(flex_A^2 \rightarrow \tau_{A,B}(flex_B^2)) \wedge \neg(flex_A^2 \leftarrow \tau_{A,B}(flex_B^2))}{A : \mathbf{comp}(B : flex_B^2) \cap A : flex_A^2} (ED-Co)$$

Alice says to Bob that they are in compatibility and makes a proposal $flex_A^2$.

$$\frac{*(B, A) \quad B : flex_B^2 \quad A : \mathbf{comp}(B : flex_B^2) \quad A : flex_A^2}{Negotiate(B, A)} (N)$$

Bob receives $flex_A^2$ and he makes a weakening or a changing action because Alice said they are not in agreement nor in relative disagreement. Suppose Bob changes her CAF by:

$$\frac{flex_B^2 \quad \neg(stub_B \leftrightarrow flex_B^2) \quad \neg(flex_B^2 \rightarrow flex_B^3) \quad \neg(flex_B^3 \rightarrow flex_B^2)}{flex_B^3} (C)$$

Bob tests the negotiation relation by:

$$\frac{A : \mathbf{comp}(B : flex_B^2) \cap A : flex_A^2 \quad (flex_B^3 \rightarrow \tau_{B,A}(flex_A^2)) \wedge \neg(flex_B^3 \leftarrow \tau_{B,A}(flex_A^2))}{B : \mathbf{relDis}(A : flex_A^2) \cap B : flex_B^3} (Co-RD)$$

Bob says to Alice that they are in relative disagreement and makes a proposal $flex_B^3$.

The system continues the MN by:

$$\frac{*(A, B) \quad A : flex_A^2 \quad B : \mathbf{relDis}(A : flex_A^2) \quad B : flex_B^3}{Negotiate(A, B)} (N)$$

Alice receives $flex_B^3$ and she cannot to make a weakening or a changing action because Bob said they are in relative disagreement. Alice accepts the proposal of Bob by:

$$\frac{B : \mathbf{relDis}(A : flex_A^2) \cap B : flex_B^3}{A : \mathbf{agree}(B : flex_B^3) \cap A : \tau_{A,B}(flex_A^2)} (RD-Ag)$$

Alice says to Bob that they are in agreement and that they have a common angle that is $flex_B^3$.

The system closes the MN by:

$$\frac{*(B, A) \quad B : flex_B^3 \quad A : \mathbf{agree}(B : flex_B^3)}{Agreement(B, A)} (A)$$

with a positive outcome, $flex_B^3$.

In Figure 16 we show the message passing flow between Alice and Bob and the changes of the EGG/YOLK configurations.

The MN results in a path, showed in Figure 17, from node 2 to node 39 of the graph in Figure 15. \square

4 Related Work

The Meaning Negotiation problem has reached large attention in the Artificial Intelligence community. Two are the most general approaches to the problem of finding a shared knowledge from many different and possibly inconsistent ones. The first way to model the MN process is by viewing it as a conflict resolution. The participants of a negotiation litigate about how to share something and they may disagree in many ways by Hunter and Summerton (2006).

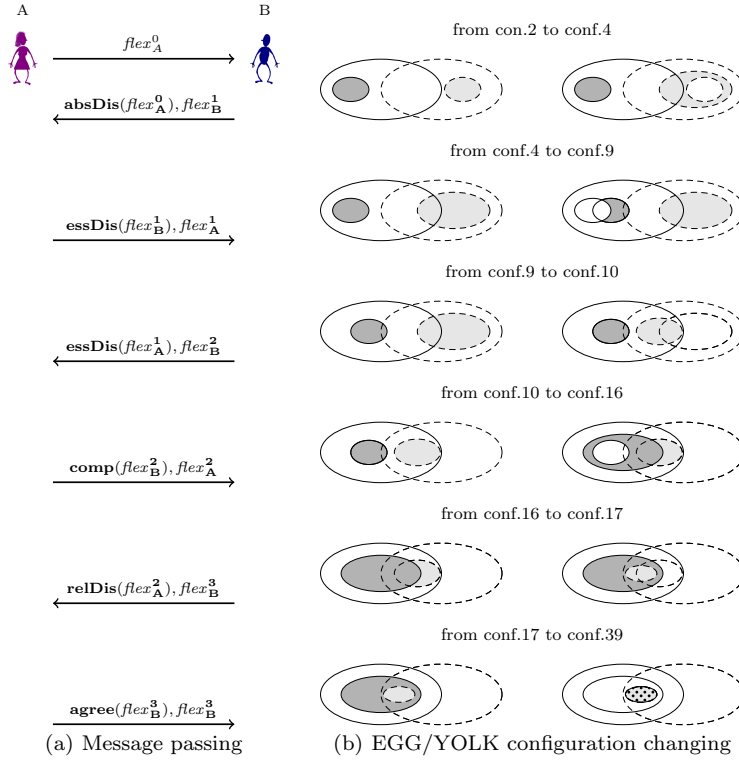


Fig. 16 A MN scenario between Alice and Bob with consistent stubbornness knowledge: the message passing flow (a) and the changes of their CAFs (b). White yolks represent the precedent proposal of the agent and the dotted gray yolk is the positive outcome of the scenario.

Argumentation theory, or argumentation, is the interdisciplinary study of how humans should, can, and do reach conclusions through logical reasoning, that is, claims based, soundly or not, on premises. It includes the arts and sciences of civil debate, dialogue, conversation, and persuasion. It studies rules of inference, logic, and procedural rules in both artificial and real world settings.

Argumentation includes debate and negotiation which are concerned with reaching mutually acceptable conclusions (Kraus et al, 1998; Parsons et al, 1998; Atkinson et al, 2005; Schroeder and Schweimeier, 2002). It also encompasses eristic dialog, the branch of social debate in which victory over an opponent is the primary goal. This art and science is often the means by which people protect their beliefs or self-interests in rational dialogue, in common parlance, and during the process of arguing. The main approaches to the Argumentation theory are: the pragma-dialectical theory and the argumentative schemes.

In pragma-dialectical theory, the argumentation is viewed as a critical discussion about the resolution of a conflicts. In this ideal model of a critical discussion, four discussion stages are distinguished that the discussion parties have to go through to resolve their difference of opinion (see (van Eemeren and Grootendorst, 1984) pp.85-88; (van Eemeren and Grootendorst, 1992), pp.34-35; (van Eemeren and Grootendorst, 2004), pp.59-62):

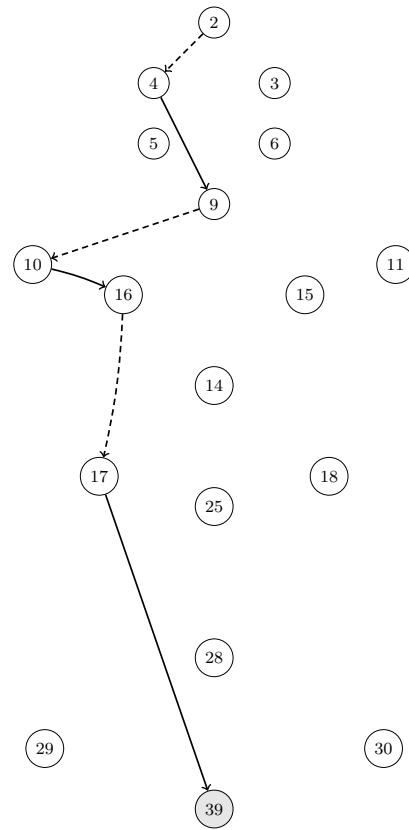


Fig. 17 The MN path of the Alice and Bob message passing in Figure 16.

1. the confrontation stage: the interlocutors establish that they have a difference of opinion;
2. opening stage: they decide to resolve this difference of opinion. The interlocutors determine their points of departure: they agree upon the rules of the discussion and establish which propositions they can use in their argumentation;
3. argumentation stage: the protagonist defends his/her standpoint by putting forward arguments to counter the antagonists objections or doubt;
4. concluding stage: the discussion parties evaluate to what extent their initial difference of opinion has been resolved and in whose favor.

The ideal model stipulates ten rules (see (van Eemeren, 2002), pp.182-183) that apply to an argumentative discussion. Violations of the discussion rules are said to frustrate the reasonable resolution of the difference of opinion and they are therefore considered as fallacies.

The representation of *Argumentative schemes* constitutes one of the central topics in current argumentation theory and they represent common patterns of reasoning used in everyday conversational discourse. Important contributions to the study of argument schemes have been made by Douglas Walton (Douglas, 1996; Prakken et al, 2003; Douglas, 2005; Douglas et al, 2008; Prakken et al, 2003). As considered by him,

argument schemes technically have the form of an inference rule: an argument scheme has a set of premises and a conclusion.

The argumentation schemes approach is based upon the Toulmin model of the argumentation (Toulmin, 2003).

The process of resolving conflicts between agents by argumentation involves not only a negotiation dialogue, but also a *persuasion* one (Walton and Krabbe, 1995). The participants in a negotiation by argumentation propose arguments to the opponents and make counterproposals in two ways: by rebutting and or by undercutting the proposals of the opponents. Rebuttal of a rule claiming c , is made by a rule in which the claim is the negation of c . A rule r undercuts a rule r' if the claim of r is the negation of some of the premises of r' .

When no undercut and rebuttal rules are available, an agent can accept the argument posted by someone else in the system in two ways (Dung et al, 2007):

- *skeptical*: the argument is acceptable until somebody else claims the contrary;
- *credulous*: the argument is wholeheartedly accepted.

In (Dung, 1995) the author explores the mechanisms humans use in argumentation to state the correctness, the appropriateness and the acceptability of arguments.

To persuade the opponents about the validity of the argument she proposes, the proponent has to *justify* it (Pollock, 1994, 2001; Walton, 2005; Rubinelli, September 2006; Katie Atkinson, 2004; Thakur et al, 2007) or to have its proof. Recent investigations have dealt with the problem about who has the burden of proving a claim and which argument produces a burden of proof (Farley and Freeman, 1995; Walton, 2003; Prakken et al, 2005; Oren et al, 2007; Gordon et al, 2007). In (Iván et al, 2000) a complete survey of the logical models of arguments is presented.

Argumentation Theory is largely used in legal reasoning to model the interactions according to the legal debate rules (Daskalopulu and Sergot, 1997; Gordon and Walton, 2009; Bench-Capon, 1997; Kowalski and Toni, 1996). In particular, in (Bench-Capon et al, 2005), the authors formalise an argumentation framework in order to model the definitions of *objectively* and *subjectively acceptable*, and *indefensible* argument. The definition of the above degrees of acceptance of an argument is based upon a value given to the arguments and a form of preference between them that the agents have.

In (Maudet et al, 2006), the authors present a brief survey of argumentation in multi-agent systems. It is not only brief, but rather idiosyncratic, and focuses on the areas of research of belief revision, agent communication and reasoning.

The second way to model MN is as a set of operations on the beliefs' sets of the agents involved. The scope is to construct a commonly accepted knowledge as the process of merging information becoming from different sources. The problem of how the merging has to be done was approached in two steps:

- how the different sources have inconsistent beliefs and how they are mutually reliable;
- how and when beliefs causing conflicts have to be merged into the knowledge base.

The first point was studied by the *information fusion* researchers and the second by the *belief revision* ones.

In (Grégoire and Konieczny, 2006) the author makes a survey of the contributions from the artificial intelligence research literature about logic-based information fusion. The assumption made by the early approaches were:

- Information sources are mutually independent;
- All sources exhibit the same level of importance;
- The level of information importance is also constant.

The main assumption regards the completely reliance of all the information sources as in (Booth, 2006). More realistic approaches suppose that the information sources are not equally reliable and that some source is preferred with respect to the available ones. In (Grégoire, 2006) the reliability of the information sources is defined as a preference order. Another precedent approach assume a weight applied to the beliefs for each source by which they belong (Lin, 1996).

In the situations in which the information sources are equally reliable, the merging is said *non-prioritized* otherwise a degree of certainty or plausibility is given to the belief (Fermé and Rott, 2004).

When the beliefs coming from the different sources, they have to be merged in order to *minimally change* the initial knowledge base. The operation needed to add new information into a knowledge base is known as *revision* and it involves only conflicting beliefs during a negotiation process. The general approach of *maximal adjustment* is to remove the present belief causing the conflict and adding the new one. In (Benferhat et al, 2004) the author present a *disjunctive maximal adjustment* in which the belief are weighted and thus not always removed or simply added into the knowledge base.

The merging² of beliefs was defined by two operators (Liberatore and Schaerf, 1998): *majority* and *arbitration*. Both make assumptions upon the information sources. The former revises the knowledge base by belief belonging to the majority number of information sources. The latter revises the knowledge bases by the beliefs belonging to the most reliable information sources.

In (Konieczny, 2000) the author defines the postulates regulating the merging operators by assuming that there are *integrity constraints* to assure.

Thus, in a belief merging and information fusion literature, the negotiation is modeled as a two stage processes: contraction of the beliefs causing the conflict and expansions by the new knowledge (Booth, 2006). In (Zhang et al, 2004) the author define a way to formalize the negotiation process as a function and he proposes a set of postulates, similar to the AGM ones for revision for the negotiation function.

5 Conclusions

We presented a formalization of the MN problem by means of a deduction system. As we remarked in many different places of the paper, the literature has dealt with several issues of the negotiation of meaning, but what has been only partially treated is the description of the process of reaching agreement conditions.

Here, we focused upon the MN problem in terms of knowledge representation and of automatic mechanism of reaching an agreement. First, we defined a negotiating agent by two set of knowledge: stubborn and flexible. The stubborn knowledge of the agent is the unquestionable one and it represents the necessary properties to define the meaning of the set of terms the agent is negotiating. Instead, the flexible knowledge

² One can be tempted to assume that arbitration and majority operators can be fruitfully employed to solve any admissible problem of negotiation. However, negotiation is the process of reaching agreements not the underlying semantic theory about the models. Therefore, although we can model the resulting theory by the theory of belief revision, negotiation processes are out of scope in these theories.

is the representation of the properties that the agent thinks as not necessary, but can be useful, to define the negotiating terms. A negotiating agent is willing to cede with respect to non necessary properties. After the definition of an agent and of her knowledge, we defined the agreement condition as the situation in which all the agent or an acceptable part of them agree with the same proposal, i.e. when the agents consider the proposal as an acceptable common angle. Otherwise the agents are in disagreement.

We identified four ways in which agents are in disagreement: absolute, essence, relative or compatibility. The disagreement relation is binary because it depends upon the relation between the knowledge of the agents, thus, for instance, Alice may have inconsistent knowledge with respect to the knowledge of Bob (absolute disagreement) and she may have a consistent but not generalised or restricted knowledge with respect to the knowledge of Charles (compatibility).

Afterwards we defined rules for deriving streams of dialog between an arbitrary number of meaning negotiating agents by assuming that in a multiparty MN the first proposing agent behaves as a referee in an English Auction Game; and we defined a deduction system, *MND*, based upon these rules, which derives a stream of dialog that ends with an agreement (or disagreement) condition.

There are several different ways in which this investigation can be taken further, in particular by investigating the formal properties of *MND*, such as soundness and completeness. The proofs of consistency and adequacy do not fix the relation to a given semantics, which is needed for a proof of soundness and a proof of completeness. Usually, a deduction system can be proved sound and complete against a standard interpretation of the language, which is difficult to circumscribe in our case, because of the presence of the relations between agents to be represented. A standard definition of the semantics for the *MND* systems is therefore needed in front of any further investigation of the soundness and completeness properties.

We deliberately avoided to investigate the formal logical properties of the system at this stage, for the sake of clarity and readability. It shall be argument of another paper.

Our formalization of the MN process may be considered *credulous* in the sense of the Argumentation literature (see Section 4). In fact, with the rule (*RD-Ag*) an agent accepts the proposal φ even if it is not equivalent to her current angle: the accepting agent trusts in the proposing agent. As a future work, we will study the properties of credulousness and skepticism of the rules of the deduction system. Moreover, the investigation of the trustworthiness among negotiating agents is interesting because a credulous or a skeptical deduction system may be adopted depending upon the trust relation among agents: an agent may be credulous with respect to a trustworthy agent and skeptical with a non-trustworthy one.

In this paper, we assumed that agents are truthful thus they never inform the opponents about something wrongly. Fraudulent agents may try to drive the MN in a way that is in some sense optimal for themselves. It would be interesting to study the optimality and minimality of the MN outcomes and the ways, legitimate or not, that the agents use to reach optimal outcomes.

It would also be interesting to develop a decision making algorithm for those cases in which the system is decidable, in particular for finite signatures in addition to the case of competitive agents considered here. This would foster the automation both of the subjective decision process (i.e., the automation of the deduction system alone) and of the whole process per se (i.e., the definition of a procedure to establish the agreement terminal condition).

The investigation we carried out can also be extended by studying the ways in which agents can be limited to specific strategies in choosing the next action. Jointly with the definition of an algorithm for negotiating a common angle, this study can also enlarge the boundary of decidable cases. In particular, agents using some specific strategies can apply the rules in a finite number of steps even if the signature is infinite.

Finally, we envisage a further extensions of our approach to applications in information security, e.g., investigating the relationships between the MN process and the management of authorization policies in security protocols and web services.

References

- Atkinson K, Bench-Capon TJM, McBurney P (2005) Arguing about cases as practical reasoning. In: ICAIL '05: Proceedings of the 10th international conference on Artificial intelligence and law, pp 35–44
- Benameur H, Chaib-draa B, Kropf P (2002) Multi-item auctions for automatic negotiation. *Information and Software Technology* 44(5):291–301
- Bench-Capon T, Atkinson K, Chorley A (2005) Persuasion and value in legal argument. *J Logic and Computation* 15(6):1075–1097
- Bench-Capon TJM (1997) Argument in artificial intelligence and law. *Artif Intell Law* 5(4):249–261
- Benferhat S, Kaci S, Berre DL, Williams MA (2004) Weakening conflicting information for iterated revision and knowledge integration. *Artif Intell* 153(1-2):339–371, DOI <http://dx.doi.org/10.1016/j.artint.2003.08.003>
- Booth R (2006) Social contraction and belief negotiation. *Inf Fusion* 7(1):19–34, DOI <http://dx.doi.org/10.1016/j.inffus.2005.01.002>
- Daskalopulu A, Sergot M (1997) The representation of legal contracts. *AI and Society* 11(1/2):6–17
- Douglas W (ed) (1996) *Argumentation Schemes for Presumptive Reasoning*. Routledge
- Douglas W (ed) (2005) *Argumentation Methods for Artificial Intelligence in Law*. Cambridge University Press
- Douglas W, Christopher R, Fabrizio M (eds) (2008) *Argumentation Schemes*. Springer
- Dung PM (1995) On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming and n-person games. *Artificial Intelligence* 77(2):321–357, DOI [http://dx.doi.org/10.1016/0004-3702\(94\)00041-X](http://dx.doi.org/10.1016/0004-3702(94)00041-X)
- Dung PM, Mancarella P, Toni F (2007) Computing ideal sceptical argumentation. *Artif Intell* 171(10-15):642–674, DOI <http://dx.doi.org/10.1016/j.artint.2007.05.003>
- van Eemeren FH (ed) (2002) *Advances in pragma-dialectics*. Amsterdam: SicSat / Newport News, VA: Vale Press
- van Eemeren FH, Grootendorst R (eds) (1984) *Speech acts in argumentative discussions: A theoretical model for the analysis of discussions directed towards solving conflicts of opinion*. Dordrecht: Floris Publications.
- van Eemeren FH, Grootendorst R (eds) (1992) *Argumentation, communication, and fallacies : a pragma-dialectical perspective*. Erlbaum: Hillsdale, NJ.
- van Eemeren FH, Grootendorst R (eds) (2004) *A systematic theory of argumentation: The pragma-dialectical approach*. Cambridge: Cambridge University Press.
- Farley AM, Freeman K (1995) Burden of proof in legal argumentation. In: ICAIL '95: Proceedings of the 5th international conference on Artificial intelligence and law,

-
- ACM, New York, NY, USA, pp 156–164, DOI <http://doi.acm.org/10.1145/222092.222227>
- Fermé E, Rott H (2004) Revision by comparison. *Artif Intell* 157(1-2):5–47, DOI <http://dx.doi.org/10.1016/j.artint.2004.04.007>
- Gordon TF (1993) The pleadings game: formalizing procedural justice. In: *ICAIL '93: Proceedings of the 4th international conference on Artificial intelligence and law*, ACM, New York, NY, USA, pp 10–19, DOI <http://doi.acm.org/10.1145/158976.158978>
- Gordon TF, Walton D (2009) Legal reasoning with argumentation schemes. In: *ICAIL '09: Proceedings of the 12th International Conference on Artificial Intelligence and Law*, ACM, New York, NY, USA, pp 137–146, DOI <http://doi.acm.org/10.1145/1568234.1568250>
- Gordon TF, Prakken H, Walton D (2007) The carneades model of argument and burden of proof. *Artificial Intelligence* 171(10-15):875–896, DOI <http://dx.doi.org/10.1016/j.artint.2007.04.010>
- Grégoire E (2006) An unbiased approach to iterated fusion by weakening. *Inf Fusion* 7(1):35–40, DOI <http://dx.doi.org/10.1016/j.inffus.2005.01.005>
- Grégoire E, Konieczny S (2006) Logic-based approaches to information fusion. *Inf Fusion* 7(1):4–18, DOI <http://dx.doi.org/10.1016/j.inffus.2005.08.001>
- Hunter A, Summerton R (2006) A knowledge-based approach to merging information. *Knowledge-Based Systems* 19(8):647–674, DOI [DOI:10.1016/j.knosys.2006.05.007](http://dx.doi.org/10.1016/j.knosys.2006.05.007), URL <http://www.sciencedirect.com/science/article/B6V0P-4KCXG2C-1/2/2929165993f89db519bcb86f9af85ff8>
- Iván CC, Maguitman AG, Loui RP (2000) Logical models of argument. *ACM Comput Surv* 32(4):337–383, DOI <http://doi.acm.org/10.1145/371578.371581>
- Kambe S (1995) Bargaining with imperfect commitment. *Games and Economic Behavior* 28(2):217–237
- Katie Atkinson PM Trevor Bench-Capon (2004) Justifying practical reasoning. In: *Proceedings of the Fourth Workshop on Computational Models of Natural Argument (CMNA'04)*, pp 87–90
- Konieczny S (2000) On the difference between merging knowledge bases and combining them. In: *KR*, pp 135–144
- Kowalski RA, Toni F (1996) Abstract argumentation. *Artificial Intelligence and Law* 4:275–296
- Kraus S, Sycara K, Evenchik A (1998) Reaching agreements through argumentation: a logical model and implementation. *Artificial Intelligence* 104(1-2):1–69
- Lehmann F, Cohn AG (1994) The egg/yolk reliability hierarchy: semantic data integration using sorts with prototypes. In: *Proc. of CIKM'94*, ACM, pp 272–279, DOI <http://doi.acm.org/10.1145/191246.191293>
- Liberatore P, Schaerf M (1998) Arbitration (or how to merge knowledge bases). *IEEE Trans on Knowl and Data Eng* 10(1):76–90, DOI <http://dx.doi.org/10.1109/69.667090>
- Lin J (1996) Integration of weighted knowledge bases. *Artificial Intelligence* 83(2):363–378, DOI [DOI:10.1016/0004-3702\(95\)00019-4](http://dx.doi.org/10.1016/0004-3702(95)00019-4), URL <http://www.sciencedirect.com/science/article/B6TYF-3VTJGS5-7/2/e443b9d18837f65ee41c70d43f990486>
- Lomuscio AR, Wooldridge M, Jennings NR (2003) A classification scheme for negotiation in electronic commerce. *Group Decision and Negotiation* 12:31–56, URL <http://dx.doi.org/10.1023/A:1022232410606>, [10.1023/A:1022232410606](http://dx.doi.org/10.1023/A:1022232410606)

- Maudet N, Parsons S, Rahwan I (2006) Argumentation in multi-agent systems: Context and recent developments. In: Maudet N, Parsons S, Rahwan I (eds) *ArgMAS*, Springer, Lecture Notes in Computer Science, vol 4766, pp 1–16
- Meyer JJ, van der Hoek W, van Linder B (1999) A logical approach to the dynamics of commitments. *Artificial Intelligence* 113(1-2):1–40, DOI [http://dx.doi.org/10.1016/S0004-3702\(99\)00061-2](http://dx.doi.org/10.1016/S0004-3702(99)00061-2)
- Oren N, Norman TJ, Preece A (2007) Subjective logic and arguing with evidence. *Artificial Intelligence* 171(10-15):838–854, DOI <http://dx.doi.org/10.1016/j.artint.2007.04.006>
- Parsons S, Sierra C, Jennings N (1998) Agents that reason and negotiate by arguing. *J Logic and Computation* 8(3):261–292
- Pollock JL (1994) Justification and defeat. *Artificial Intelligence* 67(2):377–407, DOI [http://dx.doi.org/10.1016/0004-3702\(94\)90057-4](http://dx.doi.org/10.1016/0004-3702(94)90057-4)
- Pollock JL (2001) Defeasible reasoning with variable degrees of justification. *Artificial Intelligence* 133(1-2):233 – 282, DOI [DOI:10.1016/S0004-3702\(01\)00145-X](https://doi.org/10.1016/S0004-3702(01)00145-X), URL <http://www.sciencedirect.com/science/article/B6TYF-43PH56M-2/2/6b5db756204877b78c1d7dff68440fa>
- Prakken H, Reed C, Walton D (2003) Argumentation schemes and generalisations in reasoning about evidence. In: *ICAAIL '03: Proceedings of the 9th international conference on Artificial intelligence and law*, ACM Press, New York, NY, USA, pp 32–41
- Prakken H, Reed C, Walton D (2005) Dialogues about the burden of proof. In: *ICAAIL '05: Proceedings of the 10th international conference on Artificial intelligence and law*, ACM, New York, NY, USA, pp 115–124, DOI <http://doi.acm.org/10.1145/1165485.1165503>
- Rubinelli S (September 2006) Let me tell you why!. when argumentation in doctor-patient interaction makes a difference. *Argumentation* 20:353–375(23)
- Schroeder M, Schweimeier R (2002) Arguments and misunderstandings: Fuzzy unification for negotiating agents. *Electronic Notes in Theoretical Computer Science* 70(5):1–19
- Thakur S, Governatori G, Padmanabhan V, Lundström JE (2007) Dialogue games in defeasible logic. In: Orgun MA, Thornton J (eds) *Australian Conference on Artificial Intelligence*, Springer, Lecture Notes in Computer Science, vol 4830, pp 497–506
- Toulmin S (2003) *The Uses of Argument*. Cambridge University Press
- Walton D (2003) Is there a burden of questioning? *Artif Intell Law* 11(1):1–43, DOI <http://dx.doi.org/10.1023/B:ARTI.0000013333.96215.a9>
- Walton D (2005) Justification of argument schemes. *The Australasian Journal of Logic* 3:1–13, URL <http://www.philosophy.unimelb.edu.au/ajl/2005/>
- Walton D, Krabbe E (1995) *Commitment in Dialogue*. State University of New York Press
- Wurman P, Wellman M, Walsh W (2001) A parametrization of the auction design space. *Games and Economic Behavior* 35(1-2):304–338
- Zhang D, Foo N, Meyer T, Kwok R (2004) Negotiation as mutual belief revision. In: *AAAI'04: Proceedings of the 19th national conference on Artificial intelligence*, AAAI Press / The MIT Press, pp 317–322

$\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad \neg(stub_i \wedge \tau_{i,j}(\psi))}{i : \mathbf{absDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (AD-AD)$
$\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad (stub_i \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \wedge \tau_{i,j}(\psi))}{i : \mathbf{essDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (AD-ED)$
$\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{comp}(j : \psi) \cap i : flex_i^{k+1}} \quad (AD-Co)$
$\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{relDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (AD-RD)$
$\frac{j : \mathbf{absDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi))}{i : \mathbf{agree}(j : \psi) \cap i : \tau_{i,j}(\psi)} \quad (AD-Ag)$
$\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad \neg(stub_i \wedge \tau_{i,j}(\psi))}{i : \mathbf{absDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (ED-AD)$
$\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad (stub_i \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \wedge \tau_{i,j}(\psi))}{i : \mathbf{essDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (ED-ED)$
$\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{comp}(j : \psi) \cap i : flex_i^{k+1}} \quad (ED-Co)$
$\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{relDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (ED-RD)$
$\frac{j : \mathbf{essDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi))}{i : \mathbf{agree}(j : \psi) \cap i : \tau_{i,j}(\psi)} \quad (ED-Ag)$
$\frac{j : \mathbf{comp}(i : flex_i^k) \cap j : \psi \quad (stub_i \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \wedge \tau_{i,j}(\psi))}{i : \mathbf{essDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (Co-ED)$
$\frac{j : \mathbf{comp}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{comp}(j : \psi) \cap i : flex_i^{k+1}} \quad (Co-Co)$
$\frac{j : \mathbf{comp}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{relDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (Co-RD)$
$\frac{j : \mathbf{comp}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi))}{i : \mathbf{agree}(j : \psi) \cap i : \tau_{i,j}(\psi)} \quad (Co-Ag)$
$\frac{j : \mathbf{relDis}(i : \varphi) \cap j : \psi}{i : \mathbf{agree}(j : \psi) \cap i : \tau_{i,j}(\psi)} \quad (RD-Ag)$

Table 3 Rules for the following proposing agents.

$$\begin{array}{c}
\frac{* (i, j) \quad i : \varphi \quad j : \mathbf{na}(i : \varphi) \quad j : \psi \quad stub_i \leftrightarrow \varphi \quad stub_j \leftrightarrow \psi}{Disagreement(i, j)} \quad (D) \\
\\
\frac{* (i, j) \quad i : \varphi \quad j : \mathbf{agree}(i : \varphi)}{Agreement(i, j)} \quad (A) \\
\\
\frac{* (i, j) \quad i : \varphi \quad j : \mathbf{na}(i : \varphi) \quad j : \psi}{Negotiate(i, j)} \quad (N)
\end{array}$$

Table 4 1-1 MN system transition rules.

$$\begin{array}{c}
\frac{j : \mathbf{comp}(i : flex_i^k) \cap j : \psi \quad \neg(stub_i \wedge \tau_{i,j}(\psi))}{i : \mathbf{absDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (Co-AD) \\
\\
\frac{j : \mathbf{relDis}(i : flex_i^k) \cap j : \psi \quad (stub_i \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \wedge \tau_{i,j}(\psi))}{i : \mathbf{essDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (RD-ED) \\
\\
\frac{j : \mathbf{relDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{comp}(j : \psi) \cap i : flex_i^{k+1}} \quad (RD-Co) \\
\\
\frac{j : \mathbf{relDis}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{relDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (RD-RD) \\
\\
\frac{j : \mathbf{agree}(i : flex_i^k) \cap j : \psi \quad \neg(stub_i \wedge \tau_{i,j}(\psi))}{i : \mathbf{absDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (Ag-AD) \\
\\
\frac{j : \mathbf{agree}(i : flex_i^k) \cap j : \psi \quad (stub_i \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \wedge \tau_{i,j}(\psi))}{i : \mathbf{essDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (Ag-ED) \\
\\
\frac{j : \mathbf{agree}(i : flex_i^k) \cap j : \varphi \quad (flex_i^{k+1} \vee \tau_{i,j}(\psi)) \wedge \neg(flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{comp}(j : \psi) \cap i : flex_i^{k+1}} \quad (Ag-Co) \\
\\
\frac{j : \mathbf{agree}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi)) \wedge \neg(\tau_{i,j}(\psi) \rightarrow flex_i^{k+1})}{i : \mathbf{relDis}(j : \psi) \cap i : flex_i^{k+1}} \quad (Ag-RD) \\
\\
\frac{j : \mathbf{agree}(i : flex_i^k) \cap j : \psi \quad (flex_i^{k+1} \rightarrow \tau_{i,j}(\psi))}{i : \mathbf{agree}(j : \psi) \cap i : \tau_{i,j}(\psi)} \quad (Ag-Ag)
\end{array}$$

Table 5 Extension of the rules in Table 3 for the auctioneer. All these rules are violations and the rule (Ag-Ag) may be used also by negotiating agents.

$\frac{*(a, i_1, \dots, i_n) \quad a : \varphi \quad \text{for all } i \in \text{Ag}_1.i : \mathbf{agree}(a : \varphi) \quad \text{for all } j \in \text{Ag}_2.j : \mathbf{na}(a : \varphi)}{ \text{Ag}_1 \leq \alpha \quad \text{for all } i \in \text{Ag}.i : \psi \text{ and } stub_i \leftrightarrow \psi} \quad (DD)$
$\frac{*(a, i_1, \dots, i_n) \quad a : \varphi \quad \text{for all } i \in \text{Ag}_1.i : \mathbf{agree}(a : \varphi)}{\text{for all } j \in \text{Ag}_2.j : \mathbf{na}(a : \varphi) \quad \text{Ag}_1 \geq \alpha} \quad (AA)$
$\frac{*(a, i_1, \dots, i_n) \quad a : \varphi \quad \text{for all } i \in \text{Ag}_1.i : \mathbf{agree}(a : \varphi) \quad \text{for all } j \in \text{Ag}_2.j : \mathbf{na}(a : \varphi) \quad \text{Ag}_1 \leq \alpha}{Negotiate(a, i_1, \dots, i_n)} \quad (NN)$

Table 6 1-n MN system transition rules.


Configuration	Formula
	$\neg(stub_i \wedge stub_j)$

Table 7 Configurations for inconsistent stubbornness knowledge. Agent i is identified by plain lines and agent j by dashed lines.